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New supersymmetric flux vacua with intermediate SU(2) structure

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Abstract: We find new supersymmetric four-dimensional Minkowski flux vacua of type II string theory on nilmanifolds and solvmanifolds. We extend the results of M. Graña, R. Minasian, M. Petrini, and A. Tomasiello to the case of intermediate SU(2) structures (the two internal supersymmetry parameters are neither parallel nor orthogonal). As pointed out recently by P. Koerber and D. Tsimpis, intermediate SU(2) structures are possible when one considers "mixed" orientifold projection conditions. To find our vacua, we rewrite these projection conditions in a more tractable way by introducing new variables: the projection basis. In these variables, the SUSY conditions become also much simpler to solve, and we find three new vacua. In addition, we find that these variables correspond to the SU(2) structure appearing with the dielectric pure spinors, objects introduced and discussed by R. Minasian, M. Petrini, A. Zaffaroni, and N. Halmagyi, A. Tomasiello, in the AdS/CFT context. Besides, our solutions provide some intuition on what a dynamical $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure solution could look like.

Keywords: Flux compactifications, Superstring Vacua, Differential and Algebraic Geometry.

JHEP08(2008)096

Contents

JHEP08(2008)096

1. Introduction

Flux compactifications [3] have appeared in the last few years as a promising approach to make contact between string theory and real world low energy physics. Indeed, considering non-trivial vacuum values for some supergravity fluxes on the internal manifold (on which one compactifies) has several interesting phenomenological consequences. For instance, one generates this way a potential in the effective quantum field theory, which lifts some of the moduli [4, 5]. One can also get a natural way to create hierarchies [6], and new possibilities for supersymmetry breaking [4, 5].

Previously, in order to preserve the minimal amount of supersymmetry in the low energy effective theory, one was led to consider a Calabi-Yau (CY) as the compactifications manifold [7]. The introduction of background fluxes modifies the supersymmetry conditions, leading generically to new manifolds. The general mathematical characterization of these new manifolds was given in [8, 9], where the authors rewrote the supersymmetry conditions in terms of Generalized Complex Geometry (GCG) [10, 11], and showed that the internal manifold has to be a (twisted) Generalized Calabi-Yau (GCY). An $\mathcal{N} = 1$ supergravity vacuum generically needs the existence of a pair of two globally defined non-vanishing spinors on the internal manifold. A good object to characterize this pair is then the structure group on the tangent bundle T. Indeed, in six dimensions, this pair of spinors defines either an SU(3) structure, a static SU(2) structure, or what we will call here an intermediate SU(2) structure, when respectively the spinors are parallel, orthogonal, or between the two. These different possibilities are encoded, in the GCG context, into an $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure on the bundle $T \oplus T^*$. This structure is related in GCG to the existence of a pair of compatible pure spinors. When one of the two pure spinors is closed, the manifold is said to be a Generalized Calabi-Yau.

An interesting question is to find explicit examples of these new backreacted backgrounds. A successful approach [12] has been to start from a warped CY (in the simplest case a warped T^6) with an $O3$-plane[1] and some background fluxes, and perform T-dualities to obtain new vacua on non-CY manifolds. In [1], the authors explored the possibility of using GCG to find "new" flux vacua, "new" in the sense they are neither conformal Calabi-Yau manifolds, nor T-dual to a warped T^6 with an $O3$: there are indeed some "new" vacua corresponding to nilmanifolds and solvmanifolds (twisted tori) with non trivial fluxes.

In their search for "new" four-dimensional Minkowski vacua, the authors of [1] only looked for SU(3) or static SU(2) structures, since only those seemed to be compatible with the orientifold projection. Recently in [2], it was shown that intermediate SU(2) structures are also possible when one allows a mixing of the usual SU(2) structure forms under the

[1] The compactification to four-dimensional Minkowski space-time needs the presence of space-filling orientifolds (O-planes) as sources in order to compensate the contribution of the fluxes to the energy-momentum tensor (the no-go theorem, or tadpole cancelation) [6].

JHEP08(2008)096

projection conditions. Then the last authors constructed such vacua on some GCY, starting from a warped T^6 with an $O3$ and performing some specific T-dualities.

In this paper, by first rewriting in a more tractable way the projection conditions imposed by the orientifold for intermediate SU(2) structures, we manage to find for these structures genuinely "new" four-dimensional (Minkowski) flux vacua of type II string theory with (at least) $\mathcal{N} = 1$. Note that we find them in the large volume limit with smeared sources, and for constant intermediate SU(2) structures. These vacua are not T-dual to a warped T^6 with an $O3$ because the manifolds on which we find them, the same as in [1], do not have the right isometries to perform the needed T-dualities. Furthermore, by going to the limit in which the two internal spinors are parallel or orthogonal, we find back the solutions of [1], hence providing some idea of what a generic dynamical SU(3) $\times$ SU(3) structure should look like (a dynamical structure occurs when the internal spinors, hence the structure, are varying along the manifold).

The rewriting of the orientifold projection conditions is done by introducing what we call the projection (eigen)basis, i.e. the set of structure forms which are "eigenvectors" for the projection. These forms actually define a new SU(2) structure, obtained by a rotation from the usual one. Moreover, we show that this SU(2) structure is nothing (modulo a rescaling) but the one appearing with the dielectric pure spinors. The latter are a rewriting of the GCG pure spinors, used to study the deformations of four-dimensional $\mathcal{N} = 4$ Super Yang-Mills in the context of AdS/CFT [13, 14]. As the pure spinors are much simpler when expressed with the projection basis variables, the supersymmetry conditions get much simpler. It is then easier to find solutions, which are nothing but the "new" vacua.

Here is how the paper is organized. In section 2, we give our supergravity conventions, the definitions of G-structures and our ansatz for the internal spinors, the GCG pure spinors and their properties, and finally a sum-up of the conditions a vacuum has to verify. In section 3, we derive as in [2] the projection conditions and rewrite them in a more tractable way by introducing the projection basis. Then we express the pure spinors in these variables, and explain the link with the dielectric pure spinors. Finally we give the SUSY conditions in these variables too. In section 4, after giving details on the set-up in which we are going to look for vacua, and the method used to find them, we give three solutions, among which two are T-duals. Then we study their limits to recover the solutions found in [1], and more. Finally, we look for other solutions in the specific case where there are several non completely overlapping orientifolds. In the appendix A, we give several conventions, a derivation of the SU(2) structure conditions, and the proof that some of the structure conditions imply the compatibility conditions that should be verified by the pair of GCG pure spinors to define an SU(3) $\times$ SU(3) structure. In appendix B, we give the structure conditions written in the projection basis variables, and details on the derivation of the SUSY conditions written in these variables too. In appendix C, we discuss some normalization condition related to the calibration of smeared sources. Details on the search for solutions with several orientifolds are given in appendix D.

2. Background

In this section, we give our supergravity conventions, discuss the parametrization of the

JHEP08(2008)096

internal spinors and their relation to the structure group. We also introduce pure spinors in GCG and give some of their properties. Finally, we formulate the conditions that a SUSY vacuum of type II string theory with fluxes has to satisfy, in terms of the pure spinors. Along this section, we mainly follow the conventions of [1] and [2]. Some related details are given in appendix A.

2.1 Supergravity conventions

In this paper we are interested in four-dimensional Minkowski flux vacua of type II string theory with (at least) $\mathcal{N} = 1$ supersymmetry (SUSY). Therefore we will consider type II supergravity (SUGRA) backgrounds, that are warped products of Minkowski $\mathbb{R}^{3,1}$ and of a six-dimensional compact space M_6 (assumed to be a smooth manifold). So we choose for these backgrounds the following metric:

$$ds^2_{(10)} = e^{2A(y)}\ \eta_{\mu\nu}dx^\mu dx^\nu + g_{\mu\nu}(y)dy^\mu dy^\nu\,, \tag{2.1}$$

η meaning here the diagonal Minkowski metric with signature $(-,+,+,+)$. The solutions will also have non zero background values for some of the RR and NS fluxes. Poincaré invariance in four dimensions requires the fluxes living on Minkowski to be proportional to $\mathrm{vol}_{(4)}$, the warped four-dimensional volume form. So more interestingly, we will focus on non trivial fluxes living on the internal manifold. As in [1], we define the total internal RR field F as

$$\text{IIA}: \quad F = F_0 + F_2 + F_4 + F_6\,, \tag{2.2}$$

$$\text{IIB}: \quad F = F_1 + F_3 + F_5\,, \tag{2.3}$$

with F_k the internal k-form RR field. F is related to the total ten-dimensional RR field-strength $F^{(10)}$ by

$$F^{(10)} = F + \mathrm{vol}_{(4)} \wedge \lambda(*F)\,, \tag{2.4}$$

where $*$ is the six-dimensional Hodge star, and λ is an action defined on any p-form A_p by a complete reversal of its indices

$$\lambda(A_p) = (-1)^{\frac{p(p-1)}{2}} A_p\ . \tag{2.5}$$

In order to find such solutions, one should solve the equations of motion and the Bianchi identities for the fluxes. Actually, it has been proven in [15, 16, 2] that, for the class of supergravity backgrounds we are interested in, the equations of motion for the metric and the dilaton ϕ are implied by the Bianchi identities and the ten-dimensional supersymmetry conditions, so we will solve the latter. The ten-dimensional supersymmetry conditions are the annihilation of the supersymmetry variations of the gravitino ψ_μ and the dilatino λ, given by [8]

$$\delta\psi_\mu = D_\mu\epsilon + \frac{1}{4}H_\mu\mathcal{P}\epsilon + \frac{1}{16}e^\phi\sum_n \not{F}_{2n}\gamma_\mu\mathcal{P}_n\epsilon\,, \tag{2.6}$$

$$\delta\lambda = \left(\not{\partial}\phi + \frac{1}{2}\not{H}\mathcal{P}\right)\epsilon + \frac{1}{8}e^\phi\sum_n(-1)^{2n}(5-2n)\not{F}_{2n}\mathcal{P}_n\epsilon\,, \tag{2.7}$$

with $n = 0, \ldots, 5$ for IIA and $n = \frac{1}{2}, \ldots, \frac{9}{2}$ for IIB, and $H_\mu = \frac{1}{2} H_{\mu\nu\rho}\gamma^{\nu\rho}$, H being the NSNS flux. The definitions of $\mathcal{P}$ and $\mathcal{P}_n$ are different in IIA and IIB: for IIA, $\mathcal{P} = \gamma_{11}$ and $\mathcal{P}_n = \gamma_{11}^n \sigma^1$, while for IIB, $\mathcal{P} = -\sigma^3$, $\mathcal{P}_n = \sigma^1$ for $n + \frac{1}{2}$ even and $\mathcal{P}_n = i\sigma^2$ for $n + \frac{1}{2}$ odd. The two Majorana-Weyl supersymmetry parameters of type II supergravity are arranged in the doublet $\epsilon = (\epsilon^1, \epsilon^2)$.

Because of the product structure of the solution (2.1), the Lorentz group is broken to $\mathrm{SO}(1,3) \times \mathrm{SO}(6)$ and the supersymmetry parameters ϵ^i should be decomposed accordingly. This means there should be on the compact manifold M_6 a set of independent globally defined and non-vanishing spinors noted η^i_a on which one can expand the ϵ^i as

$$\begin{aligned} \epsilon^1 &= \zeta^1 \otimes \sum_a \alpha^1_a \eta^1_a + c.c. \,, \\ \epsilon^2 &= \zeta^2 \otimes \sum_a \alpha^2_a \eta^2_a + c.c. \,. \end{aligned} \tag{2.8}$$

In this formulation, the ζ^i are the four-dimensional SUSY parameters, and the decomposition on the six-dimensional (internal) spinors can be seen from the four-dimensional point of view as internal degrees of freedom of the ζ^i. Hence, the number $\mathcal{N}$ of four-dimensional SUSYs is increased by one for each non-zero α^i_a with the corresponding internal spinor η^i_a being a Killing spinor for the SUSY conditions. So, to get at least a $\mathcal{N} = 1$ vacuum as we want, one needs at least a pair (η^1, η^2) of globally defined non-vanishing internal spinors that satisfy the SUSY conditions (and for $\mathcal{N} = 1$ one also needs $\zeta^1 = \zeta^2$). Let us now see how to parametrize this pair of internal spinors, and their relations with the G-structures one can define on the manifold.

2.2 Internal spinors and G-structures variables

A manifold M is said to admit a G-structure when its structure group is reduced to the subgroup G. The reduction is associated to the existence on the manifold of globally defined spinors. Here we are interested in SU(3) and SU(2) structures in six-dimensions.

An SU(3) structure is defined by a globally defined non-vanishing spinor η_+. In six dimensions, this spinor is a Weyl spinor so it has definite chirality. Here we take η_+ of positive chirality and of unitary norm. Complex conjugation acts as $(\eta_+)^* = \eta_-$. A G-structure is equivalently defined in terms of G-invariant no-where vanishing globally defined forms. These can be obtained as bilinears of the globally defined spinors. For an SU(3) structure, one can define a holomorphic three-form Ω_3 and a Kähler form J given by[2]

$$\begin{aligned} \Omega_{\mu\nu\rho} &= -i\eta_-^\dagger \gamma_{\mu\nu\rho} \eta_+ \,, \\ J_{\mu\nu} &= -i\eta_+^\dagger \gamma_{\mu\nu} \eta_+ \,, \end{aligned} \tag{2.9}$$

satisfying the structure conditions

$$J \wedge \Omega_3 = 0 \qquad \frac{4}{3} J^3 = i\Omega_3 \wedge \overline{\Omega}_3 \neq 0 \,. \tag{2.10}$$

[2]For both SU(3) and SU(2) structures, the holomorphicity of forms is defined with respect to the almost complex structure given in subsection 4.1. The indices $\mu,\ \nu,\ \rho$ are real.

Similarly, an SU(2) structure is defined by two orthogonal globally defined spinors η_+ and χ_+ (we take them of unitary norm). In terms of invariant forms, an SU(2) structure is given by a holomorphic one-form z (we take $||z||^2 = 2$), a real two-form j and a holomorphic two-form Ω_2 given by

$$\begin{aligned} z_\mu &= \eta_-^\dagger \gamma_\mu \chi_+ \,, \\ j_{\mu\nu} &= -i\eta_+^\dagger \gamma_{\mu\nu} \eta_+ + i\chi_+^\dagger \gamma_{\mu\nu} \chi_+ \,, \\ \Omega_{\mu\nu} &= \eta_-^\dagger \gamma_{\mu\nu} \chi_- \,, \end{aligned} \tag{2.11}$$

satisfying the following structure conditions

$$j^2 = \frac{1}{2}\, \Omega_2 \wedge \overline{\Omega}_2 \neq 0 \,, \tag{2.12}$$

$$j \wedge \Omega_2 = 0 \,, \qquad \Omega_2 \wedge \Omega_2 = 0 \,, \tag{2.13}$$

$$z \llcorner \Omega_2 = 0 \,, \qquad z \llcorner j = 0 \,. \tag{2.14}$$

where the definition of the contraction $\llcorner$ is given in appendix A.1. We give one possible derivation of these structure conditions in appendix A.2.

Note that it is possible to rewrite the spinor χ_+ as

$$\chi_+ = \frac{1}{2} z \eta_- \,. \tag{2.15}$$

The SU(2) structure is naturally embedded in the SU(3) structure defined by η_+:

$$J = j + \frac{i}{2} z \wedge \overline{z} \,, \qquad \Omega_3 = z \wedge \Omega_2 \,, \tag{2.16}$$

and one then has the reverse relations

$$j = J - \frac{i}{2} z \wedge \overline{z} \,, \qquad \Omega_2 = \frac{1}{2} \overline{z} \llcorner \Omega_3 \,. \tag{2.17}$$

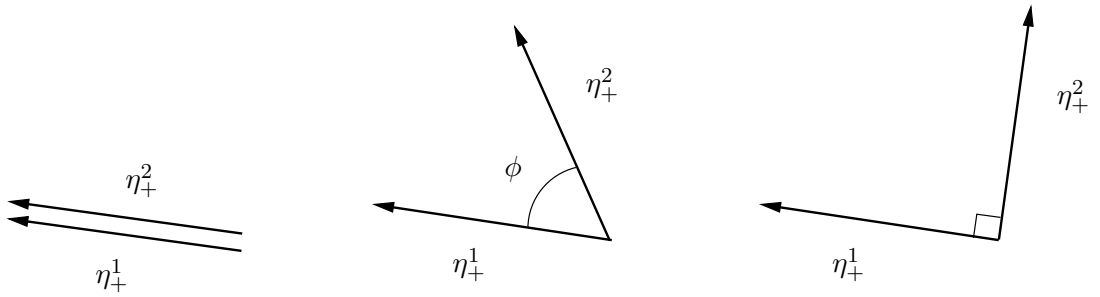
Let us consider now a pair of globally defined non-vanishing internal spinors, η_+^1 and η_+^2, corresponding to the internal components of the supersymmetry parameters. We choose to parametrize them this way (always possible):

$$\begin{aligned} \eta_+^1 &= a \eta_+ \,, \\ \eta_+^2 &= b \left(k_{||} \eta_+ + k_\perp \frac{z \eta_-}{2} \right) \,. \end{aligned} \tag{2.18}$$

η_+ and $\chi_+ = \frac{1}{2} z \eta_-$ in (2.18) define an SU(2) structure in the way explained before. $k_{||}$ is real and $0 \leq k_{||} \leq 1$, $k_\perp = \sqrt{1 - k_{||}^2}$. a and b are never-vanishing complex numbers related to the norms of the spinors η_+^i:

$$||\eta_+^1|| = |a| \,, \qquad ||\eta_+^2|| = |b| \,. \tag{2.19}$$

In the rest of the paper we will always take $|a| = |b|$, so that $||\eta_+^1|| = ||\eta_+^2||$. As we will see later, this condition is implied by the orientifold projection. The relative phases of the



SU(3) structure: $k_{||} = 1,\ k_\perp = 0$

Intermediate SU(2) structure: $k_{||} \neq 0,\ k_\perp \neq 0$

Static SU(2) structure: $k_{||} = 0,\ k_\perp = 1$

Figure 1: The different structures

spinors can be fixed by introducing $e^{i\theta} = \frac{b}{a}$ and imposing $b = \overline{a}$. The remaining freedom is then only in θ and $|a|$.

Depending on the values of the parameters $k_{||}$ and $k_\perp$, one can define from these spinors different G-structures on the internal manifold. If one takes $k_\perp = 0$, the η_+^i become parallel, hence there is only one globally defined non-vanishing spinor, and this corresponds to an SU(3) structure. When $k_\perp \neq 0$, the two spinors are genuinely independent, and so we get an SU(2) structure [17]. In the particular case $k_{||} = 0$, i.e. $k_\perp = 1$, the spinors are orthogonal, and this corresponds to what is called in the literature a static SU(2) structure. In the intermediate case ($k_{||} \neq 0$, $k_\perp \neq 0$), we have what is sometimes called a dynamical SU(2) structure, in reference to the fact these coefficients could change when we move on the manifold. We prefer to call it an intermediate SU(2) structure, because these coefficients can also be constant but still non-zero (and then the structure is not properly speaking dynamical).

It is clear that $k_{||}$ and $k_\perp$ can be related to the "angle" between the spinors. We can introduce the angle ϕ

$$k_{||} = \cos(\phi),\ k_\perp = \sin(\phi),\ 0 \leq \phi \leq \frac{\pi}{2}\,, \tag{2.20}$$

and we get for the different structures the pictures given in figure 1.

As a comparison to (2.16), one can work out the embedding of the defined SU(2) structure in the SU(3) structure defined by $\frac{\eta_+^2}{||\eta_+^2||}$ ($\tilde{J}$ and $\tilde{\Omega}_3$). It is given by the previous U(1) parameter ϕ [17]:

$$\tilde{J} = \cos(2\phi) j + \frac{i}{2} z \wedge \overline{z} + \sin(2\phi)\mathrm{Re}(\Omega_2)\,, \tag{2.21}$$

$$\tilde{\Omega}_3 = -\sin(2\phi) z \wedge j + z \wedge (\cos(2\phi)\mathrm{Re}(\Omega_2) + i\mathrm{Im}(\Omega_2))\ . \tag{2.22}$$

2.3 Pure spinors of GCG and properties

To solve the SUSY conditions, rather than using Killing spinors methods or G-structures tools, we will use the formalism of Generalized Complex Geometry (GCG). In Generalized

JHEP08(2008)096

Complex Geometry, given a manifold M_d of real dimension d, one considers the bundle $T\oplus T^*$, whose sections are generalized vectors (sums of a vector and a 1-form). For a review on GCG, see for instance [1] or the original works [10] and [11]. In this paper we will be interested in the spinors on $T\oplus T^*$. These are Majorana-Weyl Cliff(d,d) spinors, and locally they can be seen as polyforms: sums of even/odd differential forms, which correspond to positive/negative chirality spinors. A Cliff(d,d) spinor is pure if it is annihilated by half of the Cliff(d,d) gamma matrices. Such pure spinors can be obtained as tensor products of Cliff(d) spinors, since bispinors are isomorphic to forms via the Clifford map

$$C=\sum_k \frac{1}{k!}C^{(k)}_{i_1\dots i_k}dx^{i_1}\wedge\dots\wedge dx^{i_k} \quad \leftrightarrow \quad C=\sum_k \frac{1}{k!}C^{(k)}_{i_1\dots i_k}\gamma^{i_1\dots i_k}\ . \tag{2.23}$$

In the supergravity context, it is therefore natural to define the Cliff$(6,6)$ pure spinors as a bi-product of the internal supersymmetry parameters

$$\begin{aligned}\Phi_+ &= \eta^1_+\otimes\eta^{2\dagger}_+\,,\\ \Phi_- &= \eta^1_+\otimes\eta^{2\dagger}_-\ .\end{aligned} \tag{2.24}$$

They can be seen as polyforms via the Fierz identity

$$\eta^1_+\otimes\eta^{2\dagger}_\pm=\frac{1}{8}\sum_{k=0}^{6}\frac{1}{k!}\left(\eta^{2\dagger}_\pm\gamma_{\mu_k\dots\mu_1}\eta^1_+\right)\gamma^{\mu_1\dots\mu_k}\ . \tag{2.25}$$

The explicit expressions of the two pure spinors can then obtained [18] using the definitions of last subsection

$$\begin{aligned}\Phi_+ &= \frac{|a|^2}{8}e^{-i\theta}e^{\frac{1}{2}z\wedge\overline{z}}(k_{||}e^{-ij}-ik_\perp\Omega_2)\,,\\ \Phi_- &= -\frac{|a|^2}{8}z\wedge(k_\perp e^{-ij}+ik_{||}\Omega_2)\ .\end{aligned} \tag{2.26}$$

A pure spinor Ψ can always be written as [11]

$$\Psi=\Omega_k\wedge e^{B+i\omega} \tag{2.27}$$

where Ω_k is a holomorphic k-form, and B and ω are real two-forms. The rank k of Ω_k is called the type of the spinor. For the intermediate SU(2) structure where both $k_{||}$ and $k_\perp$ are non zero, it is possible to "exponentiate" Ω_2 and get from (2.26)

$$\begin{aligned}\Phi_+ &= \frac{|a|^2}{8}e^{-i\theta}k_{||}\ e^{\frac{1}{2}z\wedge\overline{z}-ij-i\frac{k_\perp}{k_{||}}\Omega_2}\,,\\ \Phi_- &= -\frac{|a|^2}{8}k_\perp\ z\wedge e^{-ij+i\frac{k_{||}}{k_\perp}\Omega_2}\,,\end{aligned} \tag{2.28}$$

so that the spinors have definite types: 0 and 1. In the case of the SU(3) structure limit ($k_\perp=0$), we get that pure spinors are of type 0 and 3

$$\Phi_+=\frac{|a|^2}{8}e^{-i\theta}e^{-iJ}\,,\qquad \Phi_-=-i\frac{|a|^2}{8}\Omega_3\,, \tag{2.29}$$

while in the case of the other limit, the static SU(2) structure ($k_{||} = 0$), the types are 1 and 2:

$$\Phi_+ = -i\frac{|a|^2}{8}e^{-i\theta}\Omega_2 \wedge e^{\frac{1}{2}z\wedge\bar{z}}\,, \qquad \Phi_- = -\frac{|a|^2}{8}z \wedge e^{-ij}\ . \tag{2.30}$$

Two pure spinors are said to be compatible if they have three common annihilators. This can be rephrased in a set of compatibility conditions the spinors must satisfy. We introduce the Mukai pairing for two polyforms Ψ_i:

$$\langle \Psi_1, \Psi_2 \rangle = (\Psi_1 \wedge \lambda(\Psi_2))_{\rm top}\ , \tag{2.31}$$

where top means the top-form, and λ has been defined in (2.5). It is also useful to recall the action of a generalized vector $X = (x, y) \in T \oplus T^*$ on a polyform

$$X \cdot \Psi_i = x \llcorner \Psi_i + y \wedge \Psi_i\ . \tag{2.32}$$

Then the compatibility conditions of two pure spinors Φ_1 and Φ_2 read

$$\left\langle \Phi_1, \overline{\Phi}_1 \right\rangle = \left\langle \Phi_2, \overline{\Phi}_2 \right\rangle \neq 0\,, \tag{2.33}$$

$$\langle \Phi_1, X \cdot \Phi_2 \rangle = \left\langle \overline{\Phi}_1, X \cdot \Phi_2 \right\rangle = 0,\ \forall\ X\ \in\ T \oplus T^*\ . \tag{2.34}$$

A pair of compatible pure spinors defines an $\mathrm{SU}(3)\times\mathrm{SU}(3)$ structure on $T\oplus T^*$. Depending on the relation between the spinors $\eta_+^{1,2}$, this translates on T into the SU(3), static SU(2) or intermediate SU(2) structures discussed above. So the formalism of GCG allows to give a unified characterization of the topological properties a $\mathcal{N} = 1$ vacuum has to satisfy: it must admit an $\mathrm{SU}(3)\times\mathrm{SU}(3)$ structure on $T\oplus T^*$. And so to satisfy this condition, we will verify that our vacua admit a pair of compatible pure spinors. One can actually show (see appendix A.3) that the "wedge" structure conditions (2.10), or (2.12) and (2.13), imply the compatibility conditions in any of the three cases, so one can verify that these conditions are satisfied, instead of the compatibility ones.

2.4 Conditions for a SUSY vacuum

An $\mathcal{N} = 1$ vacuum described in subsection 2.1 should satisfy the SUSY conditions, the equations of motion (e.o.m.) and the Bianchi identities (BI) for the fluxes. In [8], the SUSY conditions given in (2.6) and (2.7) were rewritten as differential conditions on the pure spinors:

$$(d - H\wedge)(e^{2A-\phi}\Phi_1) = 0\,, \tag{2.35}$$

$$(d - H\wedge)(e^{A-\phi}\mathrm{Re}(\Phi_2)) = 0\,, \tag{2.36}$$

$$(d - H\wedge)(e^{3A-\phi}\mathrm{Im}(\Phi_2)) = \frac{e^{4A}}{8} * \lambda(F)\,, \tag{2.37}$$

with λ defined in (2.5), and with

$$\Phi_1 = \Phi_\pm \qquad \Phi_2 = \Phi_\mp\,, \tag{2.38}$$

for IIA/IIB (upper/lower) (conventions of [1]). These conditions generalize the Calabi-Yau condition for fluxless compactifications. Indeed, the first of these equations implies

that one of the two pure spinors (the one with the same parity as the RR fields) must be twisted (because of the $-H\wedge$) conformally closed. A manifold admitting a twisted closed pure spinor is a twisted Generalized Calabi-Yau (GCY, see the precise definition in [10, 11] or [1]). So we will look for vacua on such manifolds.

The e.o.m of the fluxes read

$$(d + H\wedge)(e^{4A} * F) = 0\,, \qquad d(e^{4A-2\phi} * H) = \mp e^{4A} \sum_n F_n \wedge *F_{n+2}\,, \tag{2.39}$$

with the upper/lower sign for IIA/IIB. The BI (we assume no NS source) are

$$(d - H\wedge)F = \delta(\text{source})\,, \qquad dH = 0\ . \tag{2.40}$$

Here δ(source) is the charge density of the allowed sources: these are space-filling D-branes or orientifold planes (O-planes). In compactification to four-dimensional Minkowski, the trace of the energy-momentum tensor must be zero. This is the tadpole cancelation condition or no-go theorem [6]. Then O-planes are needed since they are the only known sources with a negative charge, that can therefore cancel the flux contribution to this trace. As in [1], in this paper we will consider smeared sources, i.e. the sources are not localized anymore. The RR BI are then assumed to be

$$(d - H\wedge)F = \sum_i Q_i V^i\,, \tag{2.41}$$

where Q_i is the source charge and V^i is (up to a sign) its internal co-volume (the co-volume of the cycle wrapped by the source). The sign of the Q_i indicates whether the source is a D-brane ($Q_i > 0$) or an O-plane ($Q_i < 0$). For more details, see section 4 and appendix C.

For intermediate SU(2) structures (for which $\frac{k_\perp}{k_{||}}$ is constant) in the large volume limit (see subsection 3.4), we will get from our SUSY conditions ((B.9) and (B.10)) that the H BI is automatically satisfied. Furthermore, for this class of compactifications, it was shown in [1] that the e.o.m. for the RR fluxes are implied by the SUSY conditions. And it was shown in [2] that the e.o.m. of H is implied by the SUSY conditions and the BI. So to sum-up, in order to find a solution, having a pair of compatible pure spinors on an GCY with at least one O-plane, we will have to verify that the SUSY conditions and the RR BI are satisfied.

3. Projection conditions and consequences

As discussed in the previous subsection, tadpole cancelation requires the inclusion in the solutions of O-plane sources. The presence of O-planes implies that the solution has to be invariant under the action of the orientifold. This imposes some projection conditions on the fields: one has to mod out by $\Omega_{\rm WS}(-1)^{F_L}\sigma$ for $O3/O7$ and $O6$, and by $\Omega_{\rm WS}\sigma$ for $O5/O9$ and $O4/O8$. $\Omega_{\rm WS}$ is a world-sheet reflection, F_L is the left-movers fermion number, and σ is an involution on the target space. The orientifold action on the pure spinors for $\mathrm{SU}(3)\times\mathrm{SU}(3)$ manifold were worked-out in [1] (see also [19]). The authors of [1] concluded that the orientifold projections are only compatible with SU(3) or static SU(2) structures.

JHEP08(2008)096

Actually, as shown in [2], intermediate SU(2) structures are also compatible with O5-, O6- and O7-planes, if one allows a mixing between the two-forms specifying the structure. In this paper, we will only consider O5- and O6-planes.

In the first subsection we repeat the derivation of the orientifold projection conditions of [2] for O5- and O6-planes. The resulting conditions on the SU(2) structure forms (j, Ω_2 and z) appear to be not very tractable. We then show in the following subsection that it is possible to rewrite these conditions in a more tractable manner, which will allow us to find directly solutions. To do so, we introduce the projection (eigen)basis, and then write the pure spinors in these variables, and discuss their relation to the dielectric ones [13, 14]. Finally, we also give the supersymmetry conditions in the projection basis (details on the derivation are in appendix B.2), and do the same for some structure conditions in appendix B.1.

3.1 The orientifold projection

As shown in [2], the first step to derive the orientifold projection on the pure spinors is to compute those for the internal SUSY parameters. This can be done starting from the projection on the ten-dimensional SUSY spinorial parameters ϵ^i, and then reducing to the internal spinors $\eta^i_\pm$. In our conventions, we get

$$O5 : \sigma(\eta^1_\pm) = \eta^2_\pm \qquad \sigma(\eta^2_\pm) = \eta^1_\pm \,, \tag{3.1}$$

$$O6 : \sigma(\eta^1_\pm) = \eta^2_\mp \qquad \sigma(\eta^2_\pm) = \eta^1_\mp \,. \tag{3.2}$$

σ is the target space reflection in the directions transverse to the O-plane. Using the expressions for the internal spinors given in (2.18), we obtain the following projection conditions at the orientifold plane:

$$O5 : e^{i\theta} = \pm 1,\ z \perp O5 \,, \tag{3.3}$$

$$O6 : e^{i\theta} \text{ free},\ \mathrm{Re}(z) \parallel O6,\ \mathrm{Im}(z) \perp O6 \,. \tag{3.4}$$

We can reexpress the previous conditions on z in the following way:

$$\begin{aligned} O5 &: \sigma(z) = -z \,, \\ O6 &: \sigma(z) = \overline{z} \,. \end{aligned} \tag{3.5}$$

As explained in [2], if the G-structures considered are constant (we will assume so), and if we work on nil/solvmanifolds (which will be our case), these conditions are valid everywhere (not only at the orientifold plane).

Following [2], starting from the projections on the $\eta^i_\pm$, we derive the projections of the pure spinors $\Phi_\pm$, and from them those for the SU(2) structure forms (using (3.3) and (3.4)). To do this last step, one has to know that, as σ is only the reflection due to the orientifold, it can distributed on every term of a wedge product. Furthermore, $\lambda(\ldots)$ can also be distributed on wedge products of two forms, provided that one of the two forms is even

(see (A.20)). So we recover the same projection conditions on the forms as they have in [2]:[3]

$$\begin{aligned} O5: \qquad \sigma(j) &= (k_{||}^2 - k_\perp^2)j + 2k_{||}k_\perp \mathrm{Re}(\Omega_2)\,, \\ \sigma(\Omega_2) &= -k_{||}^2\Omega_2 + k_\perp^2\overline{\Omega}_2 + 2k_{||}k_\perp j\,, \end{aligned} \tag{3.6}$$

$$\begin{aligned} O6: \qquad \sigma(j) &= -(k_{||}^2 - k_\perp^2)j - 2k_{||}k_\perp \mathrm{Re}(\Omega_2)\,, \\ \sigma(\Omega_2) &= k_{||}^2\overline{\Omega}_2 - k_\perp^2\Omega_2 - 2k_{||}k_\perp j\,. \end{aligned} \tag{3.7}$$

By introducing as in [2]:

$$O5: \qquad k_{||} = \cos(\phi),\ k_\perp = \sin(\phi), \qquad 0 \le \phi \le \frac{\pi}{2}\,, \tag{3.8}$$

$$\begin{aligned} O6: \qquad k_{||} &= \cos\left(\phi + \frac{\pi}{2}\right) = -\sin(\phi), \\ k_\perp &= \sin\left(\phi + \frac{\pi}{2}\right) = \cos(\phi), \qquad -\frac{\pi}{2} \le \phi \le 0\,, \end{aligned} \tag{3.9}$$

we get in both cases the more convenient formulas:

$$\begin{aligned} \sigma(j) &= \cos(2\phi)j + \sin(2\phi)\mathrm{Re}(\Omega_2)\,, \\ \sigma(\mathrm{Re}(\Omega_2)) &= \sin(2\phi)j - \cos(2\phi)\mathrm{Re}(\Omega_2)\,, \\ \sigma(\mathrm{Im}(\Omega_2)) &= -\mathrm{Im}(\Omega_2)\ . \end{aligned} \tag{3.10}$$

3.2 The projection basis

If one is looking for solutions to the projection conditions (3.10), one will notice that they are not very tractable. A good idea is to work in the projection (eigen)basis:

$$\begin{aligned} j_{||} &= \frac{1}{2}(j + \sigma(j))\,, & j_\perp &= \frac{1}{2}(j - \sigma(j))\,, \\ \mathrm{Re}(\Omega_2)_{||} &= \frac{1}{2}(\mathrm{Re}(\Omega_2) + \sigma(\mathrm{Re}(\Omega_2)))\,, & \mathrm{Re}(\Omega_2)_\perp &= \frac{1}{2}(\mathrm{Re}(\Omega_2) - \sigma(\mathrm{Re}(\Omega_2)))\ . \end{aligned} \tag{3.11}$$

Using the property $\sigma^2 = 1$ and applying it to the previous equations, we get these more tractable equations:

$$\begin{aligned} j_{||}\ (1 - \cos(2\phi)) &= \sin(2\phi)\ \mathrm{Re}(\Omega_2)_{||}\,, \\ j_\perp\ (1 + \cos(2\phi)) &= -\sin(2\phi)\ \mathrm{Re}(\Omega_2)_\perp\ . \end{aligned} \tag{3.12}$$

We also get the following equations:

$$\begin{aligned} j_{||}\ \sin(2\phi) &= (1 + \cos(2\phi))\ \mathrm{Re}(\Omega_2)_{||}\,, \\ j_\perp\ \sin(2\phi) &= -(1 - \cos(2\phi))\ \mathrm{Re}(\Omega_2)_\perp\,, \end{aligned} \tag{3.13}$$

[3] We use slightly different conventions than in [2] but actually one can start with the following general expressions which cover both articles' conventions:

$$\begin{aligned} \Phi_+ &= \frac{|a|^2}{8}e^{-i\theta}N^2 e^{\frac{1}{||z||^2}z\wedge\overline{z}}(k_{||}e^{-ij} - ik_\perp\Omega_2)\,, \\ \Phi_- &= -\frac{|a|^2}{8}\frac{\sqrt{2}}{||z||}N^2 z\wedge(k_\perp e^{-ij} + ik_{||}\Omega_2)\,, \end{aligned}$$

with $|a|$, θ, $||z||$, $||\eta_+|| = N$ constant and non-zero, and $k_{||}$, $k_\perp$ constant, and then one gets the same projection conditions.

which are equivalent to the two equations (3.12) if $k_{||}$ and $k_\perp$ are non-zero. It will be our case, so we will not use them. If we introduce (assuming that $k_{||}$ and $k_\perp$ are non-zero)

$$\begin{aligned} O5: &\qquad \gamma = \frac{k_{||}}{k_\perp}\,, \\ O6: &\qquad \gamma = -\frac{k_\perp}{k_{||}}\,, \end{aligned} \tag{3.14}$$

and $r = \mp 1$ for $O6/O5$ (upper/lower), the projection conditions become for both theories:

$$\begin{aligned} \sigma(\mathrm{Re}(z)) &= -r\ \mathrm{Re}(z)\,, \\ \sigma(\mathrm{Im}(z)) &= -\mathrm{Im}(z)\,, \\ \sigma(\mathrm{Im}(\Omega_2)) &= -\mathrm{Im}(\Omega_2)\,, \\ j_{||} &= \gamma\ \mathrm{Re}(\Omega_2)_{||}\,, \\ j_\perp &= -\frac{1}{\gamma}\ \mathrm{Re}(\Omega_2)_\perp\ . \end{aligned} \tag{3.15}$$

In this form, the projection conditions are now much more tractable.

3.3 The pure spinors and the projection basis

In this subsection, we will rewrite the pure spinors in terms of the variables of the projection basis. But before going back to the pure spinors, let us first give some useful relations (they are nothing else but a rewriting of the two last projection conditions given in (3.15)):

$$\begin{aligned} \text{IIA}: &\qquad k_{||}j_{||} + k_\perp \mathrm{Re}(\Omega_2)_{||} = 0\,, &\qquad -k_\perp j_\perp + k_{||}\mathrm{Re}(\Omega_2)_\perp = 0\,, \\ \text{IIB}: &\qquad -k_\perp j_{||} + k_{||}\mathrm{Re}(\Omega_2)_{||} = 0\,, &\qquad k_{||}j_\perp + k_\perp \mathrm{Re}(\Omega_2)_\perp = 0\ . \end{aligned} \tag{3.16}$$

These allow to write the following relations valid for both theories:

$$\begin{aligned} -\sin(\phi)j + \cos(\phi)\mathrm{Re}(\Omega_2) &= \frac{1}{\cos(\phi)}\mathrm{Re}(\Omega_2)_\perp = -\frac{1}{\sin(\phi)}j_\perp\,, \\ \cos(\phi)j + \sin(\phi)\mathrm{Re}(\Omega_2) &= \frac{1}{\sin(\phi)}\mathrm{Re}(\Omega_2)_{||} = \frac{1}{\cos(\phi)}j_{||}\ . \end{aligned} \tag{3.17}$$

These last relations (3.17) can also be found by using the definitions of $j_{||}$, $j_\perp$, $\mathrm{Re}(\Omega_2)_{||}$, and $\mathrm{Re}(\Omega_2)_\perp$. One can notice in the previous relation a rotation. We will come back to it soon.

We can now rewrite the pure spinors in (2.28) using the projection basis and the relations (3.17). The result is very simple:

$$\begin{aligned} \text{IIA}: &\qquad \Phi_+ = \frac{|a|^2}{8} e^{-i\theta} k_{||}\ e^{\frac{1}{2}z\wedge\overline{z} - \frac{i}{k_{||}k_\perp}\mathrm{Re}(\Omega_2)_\perp + \frac{k_\perp}{k_{||}}\mathrm{Im}(\Omega_2)}\,, \\ &\qquad \Phi_- = -\frac{|a|^2}{8} k_\perp\ z\wedge e^{\frac{i}{k_{||}k_\perp}\mathrm{Re}(\Omega_2)_{||} - \frac{k_{||}}{k_\perp}\mathrm{Im}(\Omega_2)}\,, \end{aligned} \tag{3.18}$$

$$\begin{aligned} \text{IIB}: &\qquad \Phi_+ = \frac{|a|^2}{8} e^{-i\theta} k_{||}\ e^{\frac{1}{2}z\wedge\overline{z} - \frac{i}{k_{||}k_\perp}\mathrm{Re}(\Omega_2)_{||} + \frac{k_\perp}{k_{||}}\mathrm{Im}(\Omega_2)}\,, \\ &\qquad \Phi_- = -\frac{|a|^2}{8} k_\perp\ z\wedge e^{\frac{i}{k_{||}k_\perp}\mathrm{Re}(\Omega_2)_\perp - \frac{k_{||}}{k_\perp}\mathrm{Im}(\Omega_2)}\ . \end{aligned} \tag{3.19}$$

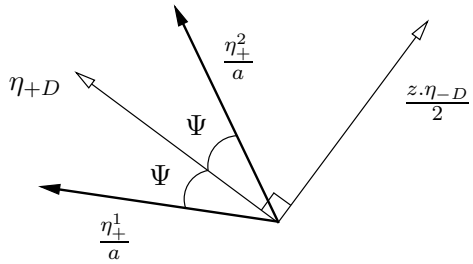


Figure 2: The different spinors and angles (with $\theta = 0$)

Recently, an alternative parametrization of the internal supersymmetry parameters, and consequently of the pure spinors, was given in [13] and further discussed in [14]

$$\begin{aligned}\eta_+^1 &= a\left(\cos(\Psi)\eta_{+D} - \sin(\Psi)\frac{z\eta_{-D}}{2}\right),\\ \eta_+^2 &= ae^{i\theta}\left(\cos(\Psi)\eta_{+D} + \sin(\Psi)\frac{z\eta_{-D}}{2}\right),\end{aligned} \tag{3.20}$$

where we still have θ as the difference of phase between η_+^2 and η_+^1, a and z are the same as before, $||\eta_{+D}|| = 1$ and Ψ is an angle such as $0 \leq \Psi \leq \frac{\pi}{4}$. This different choice was proposed in order to study deformations of four-dimensional $\mathcal{N} = 4$ Super Yang-Mills in the context of AdS/CFT. Typically those deformations should describe the near horizon geometry of some sort of dielectric branes, hence the name dielectric for the spinor η_{+D}. Note that η_{+D} is nothing else but (once the phases of the two spinors are equalled) the mean spinor between η_+^1 and η_+^2, i.e. somehow their bisector: $\eta_{+D} = \frac{1}{2a\ \cos(\Psi)}(\eta_+^1 + e^{-i\theta}\eta_+^2)$. We have the corresponding picture on figure 2. One can relate the dielectric ansatz to the previous one, (2.18), with

$$k_{||} = \cos(\phi) = \cos(2\Psi), \qquad k_\perp = \sin(\phi) = \sin(2\Psi)\,, \tag{3.21}$$

$$\eta_{+D} = \cos\left(\frac{\phi}{2}\right)\eta_+ + \sin\left(\frac{\phi}{2}\right)\frac{z\eta_-}{2}\ . \tag{3.22}$$

Working with η_{+D} and $\frac{z\eta_{-D}}{2}$ instead of η_+ and $\frac{z\eta_-}{2}$ means working with a new SU(2) structure. And this new SU(2) structure is clearly obtained by a rotation from the previous one, as one can also get by computing the relations between the SU(2) structure two-forms:

$$\begin{aligned} j_D &= k_{||}j + k_\perp \mathrm{Re}(\Omega_2)\,,\\ \mathrm{Re}(\omega_D) &= -k_\perp j + k_{||}\mathrm{Re}(\Omega_2)\,,\\ \mathrm{Im}(\omega_D) &= \mathrm{Im}(\Omega_2)\ . \end{aligned} \tag{3.23}$$

If one computes the pure spinors from (3.20) [13, 14], one gets the dielectric pure

spinors[4]

$$\Phi_+ = \frac{|a|^2}{8} e^{-i\theta} k_{||}\; e^{\frac{1}{2} z\wedge\overline{z} - \frac{i}{k_{||}} j_D + \frac{k_\perp}{k_{||}} \mathrm{Im}(\omega_D)} ,$$
$$\Phi_- = -\frac{|a|^2}{8} k_\perp\; z \wedge e^{\frac{i}{k_\perp}\mathrm{Re}(\omega_D) - \frac{k_{||}}{k_\perp}\mathrm{Im}(\omega_D)} . \tag{3.24}$$

Comparing the definitions of the two-forms (3.17) and (3.23), or the expressions for the pure spinors, (3.18), (3.19) and (3.24), we see that (for IIA/IIB)

$$\begin{aligned} j_D &= \frac{1}{k_\perp}\mathrm{Re}(\Omega_2)_{\perp/||} , \\ \mathrm{Re}(\omega_D) &= \frac{1}{k_{||}}\mathrm{Re}(\Omega_2)_{||/\perp} , \\ \mathrm{Im}(\omega_D) &= \mathrm{Im}(\Omega_2) . \end{aligned} \tag{3.25}$$

Thus the dielectric SU(2) structure variables are nothing but the eigenbasis of the orientifold projection (modulo a rescaling) ! Actually, this can be easily understood from the transformation properties of η_{+D} under the orientifold projection[5]

$$\begin{aligned} O6 &: \sigma(\eta_{\pm D}) = \eta_{\mp D} , \\ O5 &: \sigma(\eta_{\pm D}) = e^{i\theta}\eta_{\pm D} . \end{aligned} \tag{3.26}$$

Then the SU(2) bilinears constructed from it will get at most a phase and a conjugation when being applied σ, hence the three real two-forms j_D, $\mathrm{Re}(\omega_D)$ and $\mathrm{Im}(\omega_D)$ are in the projection eigenbasis, as given by (3.25). Note that these relations between those variables is a way to understand the rotation that gives the projection basis, as mentioned after (3.17).

Beside providing a tractable basis to solve the orientifold projection conditions, the dielectric variables/projection basis lead to simpler expressions of the pure spinors and so much simpler SUSY conditions (see next subsection). Hence this SU(2) structure is a much better choice to solve our problem, and we will express the equations to be solved in terms of these variables. For instance, in next subsection, we rewrite the SUSY conditions in terms of the projection basis. And in appendix B.1, we rewrite a set of SU(2) structure conditions (implying the compatibility conditions, see appendix A.3) in terms of the projection basis variables too.

3.4 SUSY equations in the projection basis

In appendix B.2.1, we give the SUSY equations (2.35), (2.36), and (2.37), expanded in forms for general expressions of the pure spinors. Here we consider a simplified version of those equations where beside the usual fixing of the parameters leading to (2.26), we choose $|a|^2 = e^A$, and go to the large volume limit, i.e. $A = 0$ and $e^\phi = g_s$ constant. This is indeed the regime in which we will look for solutions in the next section. The only remaining freedom

[4]The computation is the same as using (2.28) and introducing the dielectric SU(2) structure variables via (3.23).

[5]To get them, we recall that we have $e^{i\theta} = \pm 1$ for an $O5$, and one has to use (3.1) and (3.2).

is θ that we do not really need to fix. Moreover, we choose to look only for intermediate SU(2) structure, i.e. with $k_{||} \neq 0$ and $k_\perp \neq 0$ and constant. Taking the coefficients constant is important because it simplifies drastically the search for solutions (the SUSY conditions are much simpler), but forbids to get genuinely dynamical SU(2) structure vacua.

Using the projection basis variables and some further simplifications explained in appendices B.2.1 and B.2.2, the supersymmetry equations finally become

IIA :

$$\begin{aligned}
g_s * F_4 &= -k_\perp d(\mathrm{Im}(z)) \\
k_{||} H &= k_\perp d(\mathrm{Im}(\Omega_2)) \\
g_s * F_2 &= -k_{||} d(\mathrm{Im}(z)) \wedge \mathrm{Im}(\Omega_2) + \frac{1}{k_{||}} d(\mathrm{Re}(\Omega_2)_{||}) \wedge \mathrm{Re}(z) \\
&\quad - \frac{1}{k_\perp} H \wedge \mathrm{Im}(z) \\
g_s * F_0 &= \frac{1}{2} k_\perp d(\mathrm{Im}(z)) \wedge \mathrm{Im}(\Omega_2)^2 + \frac{1}{k_{||}} H \wedge \mathrm{Re}(z) \wedge \mathrm{Re}(\Omega_2)_{||}
\end{aligned}$$

$$\begin{aligned}
d(\mathrm{Re}(z)) &= 0 \\
d(\mathrm{Re}(\Omega_2)_\perp) &= k_{||} k_\perp \mathrm{Re}(z) \wedge d(\mathrm{Im}(z)) \\
H \wedge \mathrm{Re}(z) &= -\frac{k_\perp}{k_{||}} d(\mathrm{Im}(z) \wedge \mathrm{Re}(\Omega_2)_{||}) \,,
\end{aligned} \tag{3.27}$$

IIB :

$$\begin{aligned}
k_{||} H &= k_\perp d(\mathrm{Im}(\Omega_2)) \\
k_\perp e^{i\theta} g_s * F_3 &= d(\mathrm{Re}(\Omega_2)_{||}) \\
k_\perp e^{i\theta} g_s * F_1 &= H \wedge \mathrm{Re}(\Omega_2)_{||}
\end{aligned}$$

$$\begin{aligned}
d(\mathrm{Re}(z)) &= 0 \\
d(\mathrm{Im}(z)) &= 0 \\
\mathrm{Re}(z) \wedge H &= -\frac{k_\perp}{k_{||}} \mathrm{Im}(z) \wedge d(\mathrm{Re}(\Omega_2)_\perp) \\
\mathrm{Im}(z) \wedge H &= \frac{k_\perp}{k_{||}} \mathrm{Re}(z) \wedge d(\mathrm{Re}(\Omega_2)_\perp) \\
\mathrm{Re}(z) \wedge \mathrm{Im}(z) \wedge d(\mathrm{Re}(\Omega_2)_{||}) &= -H \wedge \mathrm{Im}(\Omega_2) \,.
\end{aligned} \tag{3.28}$$

4. Solutions

4.1 Set-up, method, and discussion

In [1], examples of four-dimensional Minkowski supersymmetric flux vacua, with a Generalized Calabi-Yau as internal manifold, were found: they correspond to nilmanifolds and solvmanifolds[6] with non trivial fluxes. As already mentioned in introduction, the analysis

[6]Nil/solvmanifolds, also known as twisted tori, can be seen as iterated fibrations of tori over other tori. They are parallelizable manifolds, namely they admit a basis of real globally defined one-forms, which we

of [1] did not take into account the possibility of an intermediate SU(2) structure on the internal manifold. Some examples of solutions with such a structure were found in [2] via T-dualities from a warped T^6 with an $O3$. In this paper, we extend the analysis of [1] and find, among nil/solvmanifolds, new vacua with intermediate SU(2) structure that cannot be T-dualized back to a warped T^6 with an $O3$.

Before describing our new solutions, we briefly sketch the method we followed. We first choose a nil/solvmanifold among the list given in [1], we specify the theory (IIA/IIB) and the internal directions of an O-plane. In this paper we will only consider O5- and O6-planes (see beginning of section 3). The orientifold projection should be compatible with the manifold algebra (see [1] for the complete list of the allowed orientifolds for each manifold). Then, one has to find a pair of compatible pure spinors on the internal manifold. The general form of the pure spinors is given in (2.26) where, in order to have an intermediate SU(2) structure, we take $k_{||} \neq 0$ and $k_\perp \neq 0$, and constant. The other coefficients in the solutions will also be taken constant. Moreover, we choose $|a|^2 = e^A$, and go to the large volume limit, i.e. where $A = 0$ and $e^\phi = g_s$ is constant.[7] We will use the set of new variables, the projection basis, which corresponds to the appropriate SU(2) structure in this problem, since many equations written in these variables get simplified (see section 3). We then solve the projection conditions (3.15) so that these pure spinors are compatible with the O-plane, and then the SU(2) structure conditions (B.2) to (B.6), and (B.7), getting automatically that the pure spinors are compatible (see appendix A.3).

This pair must satisfy the SUSY conditions, implying that one of them is closed and the manifold is thus a GCY. Still using the projection basis, we then solve the SUSY equations (3.27) or (3.28). For every solution, we can then introduce a local basis of complex one-forms $(z^1, z^2, z, \overline{z}^1, \overline{z}^2, \overline{z})$, where we identify one of them with the holomorphic one-form z of the SU(2) structure, and write the real and the holomorphic two-forms of the SU(2) structure as

$$\Omega_2 = z^1 \wedge z^2 \qquad j = \frac{i}{2}(t_1 z^1 \wedge \overline{z}^1 + t_2 z^2 \wedge \overline{z}^2 + b z^1 \wedge \overline{z}^2 - \overline{b}\overline{z}^1 \wedge z^2)\,, \tag{4.1}$$

with $b = b_r + i b_i$ and t_1, t_2, b_r, b_i real.[8] We will give our solutions in the previous form.[9]

With the almost complex structure (see footnote 2) defined trivially in the local complex basis $(z^1, z^2, z, \overline{z}^1, \overline{z}^2, \overline{z})$ by $J_\mu{}^\lambda = i\delta_\mu{}^\lambda$, $J_{\overline{\mu}}{}^{\overline{\lambda}} = -i\delta_{\overline{\mu}}{}^{\overline{\lambda}}$ ((anti)holomorphic indices),

will note e^i, $i = 1 \ldots 6$. These manifolds are group manifolds, and can be defined by their "algebra". We will use for it the following notation: $(0, 0, 0, 12, 23, 14 - 35)$, for instance, means $de^1 = de^2 = de^3 = 0$, $de^4 = e^1 \wedge e^2$, etc. with d the exterior derivative. For more details on these manifolds, see for instance [1].

[7] In [1], they give a method to localize the solutions obtained by reintroducing afterwards the warp factor. But these techniques only work for solutions with one source, while we will obtain solutions with two sources. So we will not try to get solutions in another regime than in the large volume limit, with smeared sources and constant coefficients in the solutions. As discussed for the SUSY equations, this forbids to obtain genuinely dynamical SU(2) structure solutions.

[8] Note that the choice of this basis is not unique. This freedom will appear in particular in the limits (subsection 4.3).

[9] Note that the metric we will then compute from it will be block diagonal, so the left SU(2) structure conditions, namely the contractions with z and $\overline{z}$, are clearly satisfied by these expressions.

JHEP08(2008)096

and the Kähler form defined as in (2.16), one can then compute the hermitian metric:

$$g_{\mu\overline{\nu}} = -J_{\mu}{}^{\lambda} J_{\lambda\overline{\nu}} \qquad g_{\overline{\mu}\nu} = -J_{\overline{\mu}}{}^{\overline{\lambda}} J_{\overline{\lambda}\nu} \ . \tag{4.2}$$

In this local complex basis, we obtain generically[10]

$$g = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & t_1 & b & 0 \\ 0 & 0 & 0 & \overline{b} & t_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ t_1 & \overline{b} & 0 & 0 & 0 & 0 \\ b & t_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \tag{4.3}$$

To check its definite-positiveness, one has to verify that for any μ, $g^{\mu\overline{\mu}} > 0$ (coefficients of the inverse metric), which is equivalent to $\frac{t_i}{(t_1 t_2 - |b|^2)} > 0$, $i = 1,\ 2$. Actually, the SU(2) structure condition (A.18), that the solutions verify, gives that $t_1 t_2 - |b|^2 = 1$. Hence the definite-positiveness of the metric becomes equivalent to $t_1 > 0$ and $t_2 > 0$.

The final step is to compute the RR fluxes, defined by the last SUSY equation (2.37), and to check whether they solve the Bianchi identities. Note that the metric is needed to compute the RR fluxes, because of the Hodge star. We compute the BI, and then we can determine the sources and their charges (see (2.41)). Since the sources are smeared, the BI will give us directly the directions of the co-volume V^i of the cycles wrapped by the sources. We only have to compute the correct normalization of these co-volumes. To do so, we use, as done in [1], the following identity, motivated in appendix C, and built from the calibration of the sources [20, 2]:

$$\left\langle V^i, e^{3A-\phi}\mathrm{Im}(\Phi_2) \right\rangle = \frac{1}{8 g_s}\ V \,, \tag{4.4}$$

where V is the internal volume form, defined the following way (see (2.33)):

$$\left\langle \Phi_{\pm}, \overline{\Phi}_{\pm} \right\rangle = -\frac{i}{8} ||\eta_+^1||^2 ||\eta_{\pm}^2||^2 V \,, \tag{4.5}$$

and we have $\int_{M_6} V > 0$. Note that this normalization condition is not exactly the same as in [1].[11] Once we have identified V^i, we deduce the source charge.[12] If it is negative, we deduce we have an orientifold, and we verify that the manifold and our solution are compatible with its projection.

We would like to stress that our search, on nil/solvmanifolds, for solutions with intermediate SU(2) structure is not meant to be exhaustive, our interest being to verify the possibility of having solutions of this kind that are not obtainable via T-duality. We decided

[10] Note that we give here the coefficients of the metric tensor: they are symmetric, but do not have to be real, since only the tensor has to be real. To get the metric in the real basis (e^i, $i = 1 \dots 6$), one just has to perform a change of basis.

[11] In [1], they did not have the $\frac{1}{8g_s}$ factor, that we explain in appendix C.

[12] Note that using this condition and our conventions for the Hodge star, it can be shown as in [1] that $\sum_i Q_i < 0$, and so recover the need for orientifolds as sources, because of their negative charge.

JHEP08(2008)096

Manifold	$O5$	$O6$
n 3.14	13, 15, 26, 34, 45	none
s 2.5	13, 14, 23, 24, 56	125, 136, 146, 236, 246, 345

Table 1: Directions of the possible $O5$ and $O6$ on the manifolds considered

to look at the manifolds for which non T-dual solutions with SU(3) or static SU(2) structure were found in [1] (the nilmanifold (0,0,0,12,23,14-35), noted $n3.14$, and the solvmanifold (25,-15, α 45, - α 35, 0,0), noted $s2.5$) with the intuitive hope that some intermediate SU(2) structure might be found on them, which might give back their solutions in the limits $k_{\perp/||} \to 0$. We indeed find three new solutions which we will describe in the next subsection. In subsection 4.3, we discuss their possible limits to the solutions of [1]. Note that these solutions cannot be T-dualized back to a warped T^6 with an $O3$ for the same reason as in [1]: one should T-dualize back along the internal directions chosen for the $O5$ or the $O6$. But one can see from the algebras of the manifolds that there is no isometry in these directions (an isometry direction should not appear in the algebra). In subsection 4.4, we will discuss the possibility of finding some other solutions.

Finally, let us say a word about the directions chosen for the orientifolds in our solutions. In [1] they give, for each manifold to be considered, the orientifolds compatible with the algebra (i.e. the involution σ due to the O-plane must commute with the algebra). On the two manifolds we are going to consider at first, the possible directions for the $O5$ and the $O6$ are given in table 1. Among these possibilities, we are going to look for solutions only for one set of directions on each manifold. So one could ask about the other directions. As explained in subsection 4.4 and in appendix D, one can actually consider the symmetries of the algebra to relate several possible O-planes. Furthermore, if one looks for solutions with several (not completely overlapping) O-planes, one can prove, as we do in appendix D, that it is enough to look for solutions with the sources in the directions we are going to choose.

4.2 Intermediate SU(2) solutions

We are now going to give the solutions found, with detailed steps for the first solution, and then quicker for the two others.

4.2.1 First solution

We look for IIB solutions on the nilmanifold $n3.14$ which has the following algebra: $(0,0,0,12,23,14-35)$, with an $O5$ in the 45 directions. We find the general solutions to the list of constraints (3.15), (B.2), (B.3), (B.4), (B.5), (B.6), (B.7) and (3.28). The solutions depend on the following real (constant) parameters: $b_{12}, b_{23}, b_{26}, b_{24}, b_{46}, c_{24}, c_{46}, f_1, f_3, k_{||}$ ($k_\perp$ can be replaced everywhere by $\sqrt{1-k_{||}^2}$). These parameters have to satisfy certain conditions so that the solution is genuinely one: b_{26} and $b_{24}c_{46} - c_{24}b_{46}$ have to be non-zero, f_1 or f_3 has to be non-zero, $k_{||}$ has to be nor 0 neither 1. As explained, after finding the

solutions, we expressed them as in (4.1) with for the first solution:

$$
\begin{aligned}
z &= (f_1 + if_3)e^1 + (f_3 - if_1)e^3\,,\\
z^1 &= b_{12}e^1 - b_{23}e^3 - (b_{24}+ic_{24})e^4 + \left(-i(b_{24}+ic_{24}) + \frac{k_\perp^2(b_{46}-ic_{46})(b_{24}c_{46}-c_{24}b_{46})}{b_{46}^2+k_{||}^2c_{46}^2}\right)e^5 - b_{26}e^6\,,\\
z^2 &= e^2 + \frac{(b_{46}+ic_{46})}{b_{26}}e^4 + i\frac{(b_{46}+ic_{46})}{b_{26}}e^5\,,\\
b_r &= -\frac{b_{24}b_{46}+c_{46}c_{24}k_{||}^2}{(b_{24}c_{46}-c_{24}b_{46})k_{||}k_\perp}\,, \qquad b_i = -\frac{k_{||}}{k_\perp}\,,\\
t_1 &= -\frac{b_{46}^2+c_{46}^2k_{||}^2}{b_{26}(b_{24}c_{46}-c_{24}b_{46})k_{||}k_\perp}\,, \qquad t_2 = -\frac{(b_{24}^2+c_{24}^2k_{||}^2)b_{26}}{(b_{24}c_{46}-c_{24}b_{46})k_{||}k_\perp}\,.
\end{aligned}
\tag{4.6}
$$

There is a second solution which is obtained from the first one by conjugating z and doing $e^5 \to -e^5$. The conditions on the coefficients for this second solution are the same.

As explained, the definite-positiveness of the metric is given by

$$
b_{26}(b_{24}c_{46} - c_{24}b_{46}) < 0\ . \tag{4.7}
$$

For the general solution given before, we have

$$
\begin{aligned}
H = \frac{k_\perp}{k_{||}}\Bigg(&\frac{c_{46}b_{23}-b_{46}b_{12}}{b_{26}}e^1\wedge e^2\wedge e^3 + c_{46}(e^1\wedge e^2\wedge e^6 - e^3\wedge e^4\wedge e^5)\\
&-b_{46}(e^1\wedge e^4\wedge e^5 + e^2\wedge e^3\wedge e^6)\Bigg)\,.
\end{aligned}
\tag{4.8}
$$

The general metric in the real basis can be computed with the method described previously, and its determinant $|g|$ is:

$$
\frac{(f_1^2+f_3^2)^2(-b_{46}c_{24}+c_{46}b_{24})^2(c_{46}^2+b_{46}^2)^2}{(b_{46}^2+k_{||}^2c_{46}^2)^2} \tag{4.9}
$$

(clearly non-zero). The general expression of the metric is actually quite complicated because there are many parameters, so we will not give it here. Furthermore it is difficult to compute properly its eigenvalues, and then, to use them to compute the Bianchi identities. So let us go to a simpler case, in order to show that there is at least one solution. To do so, we can make the following allowed choice of the solution's parameters:

$$
b_{12} = b_{23} = b_{46} = c_{24} = 0\ . \tag{4.10}
$$

This choice is interesting because then, the metric becomes diagonal (in the e^i basis !): its

coefficients are given by:[13]

$$g = \begin{pmatrix} f_1^2+f_3^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{b_{24}b_{26}}{c_{46}k_{||}k_\perp} & 0 & 0 & 0 & 0 \\ 0 & 0 & f_1^2+f_3^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{b_{24}c_{46}k_\perp}{b_{26}k_{||}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{b_{24}c_{46}k_\perp}{k_{||}^3 b_{26}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{k_{||}c_{46}b_{26}}{k_\perp b_{24}} \end{pmatrix} \tag{4.11}$$

Notice that with these eigenvalues, we can recheck the definite-positiveness of the metric, and we get the same condition as the one found before (4.7) with the hermitian metric: the eigenvalues are strictly positive if and only if

$$b_{26}b_{24}c_{46} < 0\ . \tag{4.12}$$

To get the Bianchi identities, we first have to be able to perform a (six-dimensional) Hodge star $*$ to get the RR fluxes (see the definitions of the fluxes in the SUSY conditions (3.28)), that is where the metric is used (see appendix A.1 for the conventions on the Hodge star). When we have a RR flux, we can then compute the Bianchi identity, and then we identify the sources obtained (see subsection 4.1) and see whether the O-planes are compatible with the manifold and the solution. Here, we get the following fluxes:

$$\begin{aligned} H &= \frac{k_\perp c_{46}}{k_{||}}\left(-e^3\wedge e^4\wedge e^5 + e^1\wedge e^2\wedge e^6\right)\ , \\ F_3 &= \frac{e^{-i\theta}|c_{46}|k_{||}^2}{g_s|b_{24}|}\Bigg(-\frac{k_\perp b_{24}c_{46}}{k_{||}^2 b_{26}}\left(e^3\wedge e^4\wedge e^6 + \frac{1}{k_{||}^2}e^1\wedge e^5\wedge e^6\right) \\ &\qquad + \frac{b_{26}}{k_\perp}\left(-\frac{1}{k_{||}^2}e^3\wedge e^5\wedge e^6 + e^1\wedge e^4\wedge e^6\right)\Bigg)\ , \\ F_1 &= \frac{e^{-i\theta}c_{46}k_{||}}{g_s|c_{46}b_{24}|}\left(-b_{26}e^1 + \frac{b_{24}c_{46}k_\perp^2}{k_{||}^2 b_{26}}e^3\right)\ . \end{aligned} \tag{4.13}$$

We then compute the Bianchi identities:

$$\begin{aligned} d(F_1) = 0\,, \qquad H\wedge F_3 = 0\,, \\ d(F_3) - H\wedge F_1 = \frac{2e^{-i\theta}|c_{46}|}{g_s|b_{24}|k_{||}^2 k_\perp}\left(\frac{k_\perp^2 b_{24}c_{46}}{b_{26}}\ e^1\wedge e^2\wedge e^3\wedge e^6 + b_{26}k_{||}^2\ e^1\wedge e^3\wedge e^4\wedge e^5\right)\ . \end{aligned} \tag{4.14}$$

We see that there is no source for F_1 (neither for F_5), which is somehow expected as we did not put any. We see that F_3 has two sources, one along the directions 45 and the other along 26. As explained in subsection 4.1, to determine their charges, we first need to

[13] Note that our convention $||z||^2 = \overline{z}^\mu z_\mu = 2$ is already implemented in the metric, by its construction from the Kähler form in which this norm appears. One can verify this point by computing this norm using either the hermitian or the real basis metric. Then, $f_1^2+f_3^2$ has nothing to do with this norm, but is only the measure related to the metric coefficients, in the real basis.

compute their co-volumes. To do so, we first compute V. From (4.5), using (2.26) for the pure spinors, and then the form (4.1) of the solutions, we get:

$$V = \frac{1}{8i} z \wedge \overline{z} \wedge \Omega_2 \wedge \overline{\Omega}_2 = -\mathrm{Re}(z^1) \wedge \mathrm{Im}(z^1) \wedge \mathrm{Re}(z^2) \wedge \mathrm{Im}(z^2) \wedge \mathrm{Re}(z) \wedge \mathrm{Im}(z) \ . \quad (4.15)$$

Note that going to the real basis given by the e^i, by replacing the z^i for each solution found, one generically gets:

$$V = C \ e^1 \wedge e^2 \wedge e^3 \wedge e^4 \wedge e^5 \wedge e^6 \ . \quad (4.16)$$

We chose the orientability conventions $\epsilon_{123456} = 1$ (see appendix A.1), so C is clearly related to $\sqrt{|g|}$, but also to the determinant[14] of the matrix allowing to go from the z^i to the e^i. So we get $C > 0$, an important point to determine the sign of the charges. Having computed V, one can determine the V^i precisely using the relation (4.4): we know already that V^i is along the transverse directions of the source, and (4.4) gives the normalization factor. Note one can rewrite (4.4), using (2.26) in the large volume limit, as

$$\begin{aligned} &\text{IIA}: && V^i \wedge \Big(\mathrm{Re}(z) \wedge \mathrm{Re}(\Omega_2)_{||} - k_{||}^2 \ \mathrm{Im}(z) \wedge \mathrm{Im}(\Omega_2) \Big) = k_{||} \ V \, , \\ &\text{IIB}: && V^i \wedge \big(\mathrm{Re}(\Omega_2)_{||} + k_\perp k_{||} \ \mathrm{Re}(z) \wedge \mathrm{Im}(z) \big) = k_\perp e^{i\theta} \ V \ . \end{aligned} \quad (4.17)$$

Finally, for the first solution, we can rewrite the BI as:

$$\begin{aligned} V^1 &= -\frac{k_\perp e^{i\theta} (f_1^2 + f_3^2) b_{26}}{k_\perp^2} \ e^1 \wedge e^2 \wedge e^3 \wedge e^6 , \\ V^2 &= -\frac{k_\perp e^{i\theta} (f_1^2 + f_3^2) b_{24} c_{46}}{k_{||}^2 b_{26}} \ e^1 \wedge e^3 \wedge e^4 \wedge e^5 \, , \\ d(F_3) - H \wedge F_1 &= -\frac{2|c_{46}|}{g_s |b_{24}| k_{||}^2 C} \left(\frac{(k_\perp b_{24} c_{46})^2}{b_{26}^2 k_{||}^2} \ V^1 + \frac{(k_{||} b_{26})^2}{k_\perp^2} \ V^2 \right) , \\ \text{with } C &= \frac{(f_1^2 + f_3^2) b_{24} c_{46}}{k_{||}^2} > 0 \, . \end{aligned} \quad (4.18)$$

So one can read directly the charges (see (2.41)) and see that $Q_1 < 0$, $Q_2 < 0$, hence we have two O-plane sources. Both are compatible with the manifold. Note that it is interesting to see this second source appearing while we only imposed the first one.

With the choice made for the parameters, the solution is (we do not display j since it is deduced easily from Ω_2 with the projection conditions):

$$\begin{aligned} \mathrm{Re}(\Omega_2)_{||} &= -\frac{b_{24} c_{46} k_\perp^2}{k_{||}^2 b_{26}} e^4 \wedge e^5 + b_{26} e^2 \wedge e^6 \, , \\ \mathrm{Re}(\Omega_2)_\perp &= b_{24} e^2 \wedge e^4 - c_{46} e^5 \wedge e^6 \, , \\ \mathrm{Im}(\Omega_2) &= \frac{b_{24}}{k_{||}^2} e^2 \wedge e^5 + c_{46} e^4 \wedge e^6 \ . \end{aligned} \quad (4.19)$$

[14] Note that checking $|g| \neq 0$ then verifies that the z^i chosen form indeed a basis. In general, we actually already have it guaranteed because of (B.7).

It is clear from this formulation that what is parallel or orthogonal under σ_{45} is also under σ_{26}. The same goes for z which only has components along e^1 and e^3. So the solution is clearly compatible with the projections of both sources.

Note that we will not find any T-dual solution to this first solution, while the two next solutions are T-duals to one another. This can be understood from table 1 since no $O6$ is compatible with $n3.14$.

4.2.2 Second solution

We proceed in the same way as for the first solution. We look for IIB solutions on the solvmanifold $s2.5$ which has the following algebra: $(25, -15, \alpha 45, -\alpha 35, 0, 0)$, $\alpha \in \mathbb{Z}$, with an $O5$ in the 13 directions. The general solution to the usual list of constraints depends on the following real (constant) parameters: $b_{25}, b_{45}, b_{24}, b_{12}, b_{23}, c_{12}, c_{23}, f_5, f_6, g_5, g_6, k_{||}$, and of course α. These parameters have to satisfy certain conditions so that the solution is genuinely one: b_{24}, $f_5 g_6 - f_6 g_5$ and $c_{23}b_{12} - c_{12}b_{23}$ have to be non-zero, $k_{||}$ has to be neither 0 nor 1, and α has to be ± 1. The solution is expressed in the usual manner with the following z^i:

$$
\begin{aligned}
z &= (f_5 + ig_5)e^5 + (f_6 + ig_6)e^6 \,, \\
z^1 &= (b_{12} + ic_{12})e^1 - (b_{23} + ic_{23})e^3 - b_{24}e^4 - b_{25}e^5 \,, \\
z^2 &= e^2 + \frac{\alpha k_\perp^2 (c_{23}b_{12} - c_{12}b_{23})^2}{(b_{12}^2 + c_{12}^2 k_{||}^2)(b_{12} + ic_{12})b_{24}} e^3 - \frac{\alpha(b_{23} + ic_{23})}{(b_{12} + ic_{12})} e^4 - \frac{1}{b_{24}} \left(b_{45} + \frac{\alpha b_{25}(b_{23} + ic_{23})}{(b_{12} + ic_{12})} \right) e^5 \,, \\
b_r &= \frac{k_\perp b_{12} c_{12}}{k_{||}(b_{12}^2 + c_{12}^2)} \,, \qquad\qquad b_i = \frac{k_\perp b_{12}^2}{k_{||}(b_{12}^2 + c_{12}^2)} \,, \\
t_1 &= -\frac{\alpha k_\perp (c_{23}b_{12} - c_{12}b_{23})}{k_{||} b_{24}(b_{12}^2 + c_{12}^2)} \,, \qquad t_2 = -\frac{b_{24}(b_{12}^2 + c_{12}^2 k_{||}^2)}{\alpha k_\perp k_{||}(c_{23}b_{12} - c_{12}b_{23})} \,.
\end{aligned}
\tag{4.20}
$$

The definite-positiveness of the metric is given by

$$
\alpha b_{24}(c_{23}b_{12} - c_{12}b_{23}) < 0 \,. \tag{4.21}
$$

For the general solution given before, and given that $\alpha^2 = 1$, we have $H = 0$, and deduce

$$
F_1 = 0 \,. \tag{4.22}
$$

The only remaining flux is then F_3. As for the first solution, the general metric is quite complicated, and it is difficult to compute its eigenvalues, so we will go to a simpler case. We just mention here its determinant, once again clearly non-zero:

$$
\frac{(b_{23}c_{12} - c_{23}b_{12})^4 (f_5 g_6 - g_5 f_6)^2}{(c_{12}^2 k_{||}^2 + b_{12}^2)^2} \,. \tag{4.23}
$$

To simplify the metric, we first choose $b_{25} = b_{45} = 0$. Then to get a diagonal metric, one would need $b_{23}b_{12} + c_{12}c_{23}k_{||}^2 = 0$, $g_6 g_5 + f_6 f_5 = 0$. We choose this stronger simplification:

$$
b_{25} = b_{45} = b_{23} = c_{12} = g_5 = f_6 = 0 \,. \tag{4.24}
$$

The metric is then:

$$g = \begin{pmatrix} -\frac{\alpha c_{23} b_{12} k_\perp}{k_{||} b_{24}} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{b_{24} b_{12}}{k_\perp k_{||} \alpha c_{23}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\alpha k_\perp c_{23}^3 k_{||}}{b_{12} b_{24}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{c_{23} k_{||} b_{24} \alpha}{b_{12} k_\perp} & 0 & 0 \\ 0 & 0 & 0 & 0 & f_5^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_6^2 \end{pmatrix} \tag{4.25}$$

We recover the definite-positiveness of the metric (coherent with (4.21)):

$$\alpha b_{24} c_{23} b_{12} < 0 \ . \tag{4.26}$$

Using the same method as before, we then get:

$$F_3 = \frac{e^{-i\theta}(-k_\perp^2 c_{23}^2 + b_{24}^2)\ |g_6|}{g_s k_\perp b_{24} |f_5|} (e^2 \wedge e^3 \wedge e^6 + \alpha\ e^1 \wedge e^4 \wedge e^6) \,, \tag{4.27}$$

$$d(F_3) = 2\frac{e^{-i\theta}(-k_\perp^2 c_{23}^2 + b_{24}^2)\ |g_6|}{g_s k_\perp b_{24} |f_5|} (e^1 \wedge e^3 \wedge e^5 \wedge e^6 - \alpha\ e^2 \wedge e^4 \wedge e^5 \wedge e^6) \ . \tag{4.28}$$

We see that F_3 has two sources, the one along 13 as expected, and we discover that a second one is then absolutely needed: one along 24. As before, we compute the co-volumes and get:

$$V^1 = -\frac{k_\perp e^{i\theta} f_5 g_6 \alpha c_{23}^2}{b_{24}}\ e^1 \wedge e^3 \wedge e^5 \wedge e^6, \ V^2 = -\frac{e^{i\theta} f_5 g_6 b_{24}}{k_\perp} e^2 \wedge e^4 \wedge e^5 \wedge e^6 \,, \tag{4.29}$$

$$d(F_3) = -\frac{2(b_{24}^2 - k_\perp^2 c_{23}^2)|g_6| c_{23}^2}{g_s |f_5| C} \left(\frac{1}{k_\perp^2 c_{23}^2} V^1 - \frac{1}{b_{24}^2} V^2 \right), \text{ with } C = f_5 g_6 \alpha c_{23}^2 > 0 \,. \tag{4.30}$$

The nature of the sources depends on the sign of their charges, which depends here on the value of the parameters. But we can clearly see that there is one O-plane and one D-brane. In both cases, the O-plane is compatible with the manifold. Note also that we clearly have $\sum_i Q_i < 0$.

The solution with the simple choice of parameters is:

$$\begin{aligned} \mathrm{Re}(\Omega_2)_{||} &= \frac{\alpha k_\perp^2 c_{23}^2}{b_{24}} e^1 \wedge e^3 + b_{24} e^2 \wedge e^4 \,, \\ \mathrm{Re}(\Omega_2)_\perp &= b_{12} e^1 \wedge e^2 - \frac{\alpha c_{23}^2 k_{||}^2}{b_{12}} e^3 \wedge e^4 \,, \\ \mathrm{Im}(\Omega_2) &= -\alpha c_{23} e^1 \wedge e^4 + c_{23} e^2 \wedge e^3 \ . \end{aligned} \tag{4.31}$$

As for the first solution, it is clear from this formulation that what is parallel or orthogonal under σ_{13} is also under σ_{24}. The same goes for z which has only components along e^5 and e^6. So the solution is clearly compatible with the projections of both sources (in case they are O-planes).

4.2.3 Third solution

We proceed as for the previous solutions. We look for IIA solutions on the solvmanifold $s2.5$ which has the following algebra: $(25, -15, \alpha 45, -\alpha 35, 0, 0)$, $\alpha \in \mathbb{Z}$, but now with an $O6$ in the 136 directions. We are going to see that this solution is T-dual to the second one, so there will be a lot of similarities between the two. The solution to the usual list of constraints depends on the following real (constant) parameters: $b_{25}, b_{45}, b_{24}, b_{12}, b_{23}, c_{12}, c_{23}, f_6, g_5, k_{||}, \alpha$ which have to satisfy almost the same conditions as the second solution does: b_{24}, $f_6 g_5$ and $c_{23}b_{12} - c_{12}b_{23}$ have to be non-zero, $k_{||}$ has to be neither 0 nor 1, and α has to be ± 1. The general solution is expressed in the usual manner with the following z^i:

$$
\begin{aligned}
z &= i g_5 e^5 + f_6 e^6 \,, \\
z^1 &= (b_{12} + i c_{12}) e^1 - (b_{23} + i c_{23}) e^3 - b_{24} e^4 - b_{25} e^5 \,, \\
z^2 &= e^2 + \frac{\alpha k_{||}^2 (c_{23} b_{12} - c_{12} b_{23})^2}{(b_{12}^2 + c_{12}^2 k_\perp^2)(b_{12} + i c_{12}) b_{24}} e^3 - \frac{\alpha (b_{23} + i c_{23})}{(b_{12} + i c_{12})} e^4 - \frac{1}{b_{24}} \left(b_{45} + \frac{\alpha b_{25} (b_{23} + i c_{23})}{(b_{12} + i c_{12})} \right) e^5 \,, \\
b_r &= -\frac{k_{||} b_{12} c_{12}}{k_\perp (b_{12}^2 + c_{12}^2)} \,, \qquad b_i = -\frac{k_{||} b_{12}^2}{k_\perp (b_{12}^2 + c_{12}^2)} \,, \\
t_1 &= \frac{\alpha k_{||} (c_{23} b_{12} - c_{12} b_{23})}{k_\perp b_{24} (b_{12}^2 + c_{12}^2)} \,, \qquad t_2 = \frac{b_{24} (b_{12}^2 + c_{12}^2 k_\perp^2)}{\alpha k_\perp k_{||} (c_{23} b_{12} - c_{12} b_{23})} \,.
\end{aligned}
\tag{4.32}
$$

The definite-positiveness of the metric is given by

$$
\alpha b_{24} (c_{23} b_{12} - c_{12} b_{23}) > 0 \,. \tag{4.33}
$$

For the general solution given before, and given that $\alpha^2 = 1$, we have $H = 0$ and $d(\mathrm{Im}(z)) = 0$. Hence we deduce

$$
\begin{aligned}
F_0 &= 0 \,, \\
F_4 &= 0 \,.
\end{aligned}
\tag{4.34}
$$

The only remaining flux is then F_2. The general metric determinant is (clearly non-zero):

$$
\frac{(b_{23} c_{12} - c_{23} b_{12})^4 g_5^2 f_6^2}{(c_{12}^2 k_\perp^2 + b_{12}^2)^2} \,. \tag{4.35}
$$

For the same reasons as for the previous solutions, we go to a simpler case. We first choose $b_{25} = b_{45} = 0$. Then to get a diagonal metric, one would need $b_{23} b_{12} + c_{12} c_{23} k_\perp^2 = 0$. We choose this stronger simplification:

$$
b_{25} = b_{45} = b_{23} = c_{12} = 0 \,. \tag{4.36}
$$

The metric is then:

$$g = \begin{pmatrix} \frac{\alpha c_{23} b_{12} k_{||}}{k_\perp b_{24}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{b_{24} b_{12}}{k_{||} k_\perp \alpha c_{23}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha k_{||} c_{23}^3 k_\perp}{b_{12} b_{24}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{c_{23} k_\perp b_{24} \alpha}{b_{12} k_{||}} & 0 & 0 \\ 0 & 0 & 0 & 0 & g_5^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_6^2 \end{pmatrix} \tag{4.37}$$

We recover the definite-positiveness of the metric (coherent with (4.33)):

$$\alpha b_{24} c_{23} b_{12} > 0 \ . \tag{4.38}$$

Using the same method as before, we then get:

$$F_2 = \frac{(-k_{||}^2 c_{23}^2 + b_{24}^2)\ |f_6|}{g_s k_{||} b_{24} f_6 |g_5|} (e^2 \wedge e^3 + \alpha\ e^1 \wedge e^4) \,, \tag{4.39}$$

$$d(F_2) = 2 \frac{(-k_{||}^2 c_{23}^2 + b_{24}^2)\ |f_6|}{g_s k_{||} b_{24} f_6 |g_5|} (e^1 \wedge e^3 \wedge e^5 - \alpha\ e^2 \wedge e^4 \wedge e^5) \ . \tag{4.40}$$

We see that F_2 has two sources, the one along 136 as expected, and we discover that a second one is then absolutely needed: one along 246. As before, we compute the co-volumes (see (4.17)) and get:

$$V^1 = \frac{k_{||} g_5 \alpha c_{23}^2}{b_{24}}\ e^1 \wedge e^3 \wedge e^5, \qquad V^2 = \frac{g_5 b_{24}}{k_{||}}\ e^2 \wedge e^4 \wedge e^5 \tag{4.41}$$

$$d(F_2) = -\frac{2(b_{24}^2 - k_{||}^2 c_{23}^2)|f_6| c_{23}^2}{g_s |g_5| C} \left(\frac{1}{k_{||}^2 c_{23}^2} V^1 - \frac{1}{b_{24}^2} V^2 \right), \quad \text{with } C = -g_5 f_6 \alpha c_{23}^2 > 0 \,. \tag{4.42}$$

The nature of the sources depends on the sign of their charges, which depends here on the value of the parameters. But we can clearly see that there is one O-plane and one D-brane. In both cases, the O-plane is compatible with the manifold. Note also that we clearly have $\sum_i Q_i < 0$.

The solution with the simple choice of parameters is:

$$\begin{aligned} \mathrm{Re}(\Omega_2)_{||} &= \frac{\alpha k_{||}^2 c_{23}^2}{b_{24}} e^1 \wedge e^3 + b_{24} e^2 \wedge e^4 \,, \\ \mathrm{Re}(\Omega_2)_\perp &= b_{12} e^1 \wedge e^2 - \frac{\alpha c_{23}^2 k_\perp^2}{b_{12}} e^3 \wedge e^4 \,, \\ \mathrm{Im}(\Omega_2) &= -\alpha c_{23} e^1 \wedge e^4 + c_{23} e^2 \wedge e^3 \ . \end{aligned} \tag{4.43}$$

It is clear from this formulation that what is parallel or orthogonal under σ_{136} is also under σ_{246}. The same goes for $\mathrm{Re}(z)$ which is along e^6 and $\mathrm{Im}(z)$ which is along e^5. So the solution is clearly compatible with the projections of both sources (in case they are O-planes).

We claimed that this solution was T-dual to the second one, with, obviously from the sources, a T-duality in the e^6 direction. Note that e^6 is a component of z. This can

	SU(3)		Int. SU(2)		Stat. SU(2)
Φ_+	0	$\longleftarrow$	0	$\dashrightarrow$	2
Φ_-	3	$\dashleftarrow$	1	$\longrightarrow$	1

Table 2: Pure spinors types and limits.

be understood the following way. In [21], they derived the T-duality rules for the GCG pure spinors, summed-up in [1]. Using these rules, and the SU(2) structure contraction properties (A.13) and (A.16), one can show easily that a T-duality in a direction given $\mathrm{Re}(z)$ or $\mathrm{Im}(z)$ is just the exchange of the two pure spinors (3.18) and (3.19), modulo a possible phase. This is because the terms in z in the pure spinors get exchanged by the T-duality. The exchange of the pure spinors modulo a phase can be summarized by for instance:

$$k_\perp \to -k_{||},\ k_{||} \to k_\perp\ . \tag{4.44}$$

Going from one theory to the other, the two pure spinors are always exchanged in the SUSY equations. So with the previous T-duality, a solution in one theory becomes a solution in the T-dual theory. Hence taking a solution in one theory, doing the change (4.44), one gets a T-dual solution in the other theory, where the T-duality has been done in $\mathrm{Re}(z)$ or $\mathrm{Im}(z)$ direction. This is exactly what happens between the second and the third solution, that is why we can say they are T-dual. Note that we also understand from (4.44) why an SU(3) structure is dual to a static SU(2) structure, as it is the case for the solutions in [1] (see next subsection).

4.3 SU(3) or static SU(2) structures limits

In [1], SU(3) or static SU(2) structure solutions were found on the manifolds we have just studied. So it is interesting to see what happens to our solutions when we take one of those two limits: it would be somehow natural to recover the solutions of [1]. It was at first the kind of intuition that led us to look for intermediate SU(2) solutions on these manifolds. To take the limit on our solutions, one has two options: taking the limit of the pure spinors, or taking the limit of the structure forms. Taking the limit of the pure spinors might not be a good idea. Indeed, we know pure spinors have different types (see subsection 2.3) for each G-structure, so there might be a problem when taking the limit. More precisely, only one of the two spinors keeps the same type in the limit, so this pure spinor might transform smoothly, while the other might not. This is summed-up in table 2, with the plain arrows indicating the smooth limits and the dashed ones indicating the limits where there might be a problem. We recover this point when considering the dielectric pure spinors expressions (3.24): when one replaces first j_D and $\mathrm{Re}(\omega_D)$ by their expressions, and then takes the limit, one does not get the correct expressions for the pure spinors. To get them right, one has to use the following prescription: first take the limit of j_D and $\mathrm{Re}(\omega_D)$, and then the limit of the expression obtained.

This prescription is more in favor of the second option: taking the limit of the structure forms, and that is what we will do. Looking at the expressions of the dielectric forms j_D

	SU(3) $k_\perp \to 0$	Static SU(2) $k_{||} \to 0$
IIA	$(\frac{1}{k_{||}})\mathrm{Re}(\Omega_2)_{||} \to \mathrm{Re}(\Omega_2)$ $\frac{1}{k_\perp}\mathrm{Re}(\Omega_2)_\perp \to j$	$(\frac{1}{k_\perp})\mathrm{Re}(\Omega_2)_\perp \to \mathrm{Re}(\Omega_2)$ $-\frac{1}{k_{||}}\mathrm{Re}(\Omega_2)_{||} \to j$
IIB	$(\frac{1}{k_{||}})\mathrm{Re}(\Omega_2)_\perp \to \mathrm{Re}(\Omega_2)$ $\frac{1}{k_\perp}\mathrm{Re}(\Omega_2)_{||} \to j$	$(\frac{1}{k_\perp})\mathrm{Re}(\Omega_2)_{||} \to \mathrm{Re}(\Omega_2)$ $-\frac{1}{k_{||}}\mathrm{Re}(\Omega_2)_\perp \to j$

Table 3: Limits of structure forms.

and ω_D in (3.23), we see that their limits give straightforwardly the forms of the limit structures. Actually, we prefer to use the projection basis $\mathrm{Re}(\Omega_2)_{||}$, $\mathrm{Re}(\Omega_2)_\perp$ and $\mathrm{Im}(\Omega_2)$, as we gave our solutions with these variables. More precisely, we are going to take the limit of $\mathrm{Im}(\Omega_2)$ and $\frac{1}{k_\ldots}\mathrm{Re}(\Omega_2)_\ldots$, where $\ldots$ stands for $||$ or $\perp$. Doing so, we also recover the forms of the limit structures, as one can see from (3.25) or (3.17). The results are given in table 3.[15] It is clear that $\frac{1}{k_\ldots}\mathrm{Re}(\Omega_2)_\ldots$ is not the best choice for taking the limit since $k_{||}$ or $k_\perp$, assumed non-zero, have to go to zero.[16] Indeed, one can see from the previous arrays that Ω_2 is always recovered smoothly while j is not recovered very easily. For instance in the case IIA and SU(3) limit, $k_\perp$ and $\mathrm{Re}(\Omega_2)_\perp$ both go to zero, and only their fraction is supposed to give back j. To get a well-defined limit, we should have a non-zero j, and so we must have in the previous example $\mathrm{Re}(\Omega_2)_\perp \sim k_\perp f_2 \to 0$, where f_2 stands for a constant real two-form. Imposing this last condition will give us the behaviour of some of our parameters. It can also sometimes lead to inconsistencies such as the volume form going to zero, and then we can say that there is no limit solution.

Here is how we will proceed. By first studying the limit to j, we get conditions on the behaviour of our parameters: some go to zero in a specific way, as just explained. Using them, we work out the limit to Ω_2 (extrapolated to Ω_3 in the SU(3) case), and manage to get the z^i of [1] solutions, noted z^i_s, by factorizing the form as they do. Then we work out completely the limit to j (extrapolated to J in the SU(3) case), and find the needed t_{is} and b_s (same notations as (4.1)) to get their solution. Finally, we verify that we have the same fluxes as they do when taking the limit on ours.

The validity of this procedure could be discussed further. In particular, we do recover the structure forms found in [1] (modulo global normalization factors) as we find maps between their parameters and ours. But there is a possible mismatch for the H flux in the static SU(2) limit, as one can see from its definition in the SUSY conditions (3.27) or (3.28). Indeed, if we did not find any H in the intermediate case, we cannot take its limit to recover an H in the static SU(2) limit, while the SUSY conditions allow for a non-trivial H in this limit. This situation will happen for our third solution, as they do

[15] Note that we recover in these limits the fact that, according to the projection conditions (3.10), j and $\mathrm{Re}(\Omega_2)$ (and $\mathrm{Im}(\Omega_2)$) of the limit structures are the projection eigenbasis.

[16] The difficulties that can occur are related to the one just explained for the pure spinors, since they both are related to the assumption of $k_{||}$ and $k_\perp$ being non-zero.

JHEP08(2008)096

find a possible H in [1] while we do not. For our second solution, this problem could also have occurred, but no H was found in [1]. Note that if there is a mismatch with H, then there is possible one with the other fluxes, as we can see from their definitions.

4.3.1 Limits of the first solution

Let us first consider the SU(3) limit of the first solution which should correspond to "Model 1" of [1] (same theory, same manifold, same orientifold(s)). Imposing that $\mathrm{Re}(\Omega_2)_{||}$ goes to zero ($\sim k_\perp$) and comparing with their J_s gives these behaviours for our parameters: $b_{12} \ll k_\perp$, $b_{23} \ll k_\perp$ and $b_{26} \sim -k_\perp$ (with a possible positive constant[17] that we will not consider for simplicity). Note that a priori in our solution b_{12} and b_{23} could be zero but b_{26} could not, so can we put it to zero? This is actually possible only when taking the limit, we forbade it when looking for solutions because we restricted ourselves to pure intermediate cases. One criteria to verify that the limit is well-defined is that the six-form volume must not go to zero. And b_{26} actually does not appear in it, as one can see from the determinant of the metric (4.9), so it is fine. So using these behaviours of our parameters and the limits given in the array, we get their Ω_{3s} and J_s with a global normalization difference. The normalization factor affects both Ω_3 and J so that the normalization condition (2.10) is still satisfied for both our limits and the forms in [1]. We decide to take this factor into account by rescaling some of the z^i_s and the t_{is} to match the one we have when taking the limit. We get:[18]

$$z^1 = (f_1 + i f_3)(e^1 - i e^3),\ z^2 = e^2 + i\tau e^6,\ z^3 = (b_{24} + i c_{24})(e^4 + i e^5),\ \text{with } \tau = i\frac{(b_{46} + i c_{46})}{(b_{24} + i c_{24})}\,, \tag{4.45}$$

$$t_1 = 1, \qquad t_2 = -\frac{1}{\tau_r}, \qquad t_3 = -\tau_r, \qquad \text{with } \tau_r = \mathrm{Re}(\tau)\,. \tag{4.46}$$

Looking at our fluxes, we get that $H \to 0$ as in [1], and we deduce that $F_1 \to 0$ when we look at the SUSY conditions (3.28). To compare the F_3, we go to the simpler case we chose for our coefficients (4.10): it gives $\mathrm{Im}(\tau) = 0$. In this case, we recover $F_1 \to 0$ when looking at its expression. Moreover, taking the limit on our F_3, we recover the solution of [1], once the t_{is} are rescaled as explained.

Let us now consider the SU(2) limit. Looking at the condition $\mathrm{Re}(\Omega_2)_\perp \to 0$, one gets at least $b_{46} \sim c_{46} \to 0$ (with a possible constant), and $b_{24} \to 0$. But this is not allowed, because the volume form would go to zero (see for instance (4.9)). So we recover the statement of [1]: there is no static SU(2) limit. Note that a T-dual on this manifold to the SU(3) limit would have been a static SU(2) structure with an $O6$. Then, the fact that there is no static SU(2) on this manifold can also be understood by the fact that there is no $O6$ compatible, according to table 1.

[17] The sign comes from the study of the C appearing in the charges computation and the definite-positiveness of the metric.

[18] Note that we have here an example of a different choice for the z^i, mentioned in subsection 4.1. So it would have been surprising to recover theirs by taking the limit of our z^i. The way we recovered their solution is a reparametrization, since we computed the two-form in the limit and then refactorized it in the way they did.

JHEP08(2008)096

4.3.2 Limits of the second solution

Let us first consider the SU(3) limit of the second solution. We mention first that no corresponding solution is mentioned in [1]. There can be several reasons for this, among them one can be that there is no solution with fluxes which is non T-dual to a warped T^6 with an $O3$. We actually do find such a solution, which should be the T-dual to the static SU(2) limit of our third solution (see next subsection). So we will use similar notations. Considering as usual $\mathrm{Re}(\Omega_2)_{||} \to 0$ ($\sim k_\perp$), we get $b_{25} \sim y_1 k_\perp$, $b_{45} \sim y_2 k_\perp$, $b_{24} \sim x k_\perp$ with y_1, y_2, x real constants. As for the previous solution, $b_{25} \to 0$, $b_{45} \to 0$ are allowed in our solution, but $b_{24} \to 0$ is not for an intermediate SU(2) structure. With the same arguments as before, it can actually be allowed in the SU(3) limit (see (4.23)). Using these behaviours of our parameters, we get:

$$\Omega_{3\ \mathrm{SU(3)}} = (b_{12}+ic_{12})\ ((f_5+ig_5)\ e^5+(f_6+ig_6)\ e^6)\wedge(e^1-\tau e^3)\wedge\left(e^2-\alpha\tau e^4-\frac{1}{x}(y_2+y_1\alpha\tau)e^5\right), \tag{4.47}$$

with $\tau = \frac{b_{23}+ic_{23}}{b_{12}+ic_{12}}$ (clearly of the same form as the static SU(2) limit of the third solution).

Let us now consider the fluxes. We get $H = 0$ and then $F_1 = 0$. In the simpler case chosen for the parameters (4.24), we get a non-trivial F_3 in the SU(3) limit:

$$F_{3\ \mathrm{SU(3)}} = \frac{e^{-i\theta}(-c_{23}^2+x^2)\ |g_6|}{g_s x|f_5|}(e^2\wedge e^3\wedge e^6+\alpha\ e^1\wedge e^4\wedge e^6)\ . \tag{4.48}$$

With this simple choice for the parameters, we have $y_1 = y_2 = 0$, so the solution obtained in the limit is compatible with the two sources appearing when computing the BI.

Let us now consider the static SU(2) limit of the second solution, which should correspond to "Model 2" in [1] (taking $\alpha = 1$). Our z is clearly the same as theirs. By imposing that $\mathrm{Re}(\Omega_2)_\perp$ goes to zero ($\sim k_{||}$) and comparing its limit with their j_s, we get these behaviours for our parameters: $b_{12} \sim -x k_{||}$, $b_{23} \sim -y k_{||}$ where x and y are real constants. It was forbidden in our solution to put these parameters to zero but when one looks closely at the volume form (see for instance (4.23)), one sees it can be allowed in the static SU(2) limit. The solution given in [1] is the following:

$$\Omega_{2s} = (e^1+i(-\tau_2^2e^2+\tau_2^1e^4+\tau_3^1e^5))\wedge\left(e^3+i\left(\tau_2^2e^4+\tau_3^2e^5+\left(2\frac{b_s}{t_{2s}}\tau_2^2+\frac{t_{1s}}{t_{2s}}\tau_2^1\right)e^2\right)\right)\ , \tag{4.49}$$

with all parameters real, and $t_{2s} = \frac{1+b_s^2}{t_{1s}}$. When taking the limit on our forms, we get the same result, with a global normalization factor difference: our $\Omega_{2\ \mathrm{static\ SU(2)}}$ and our $j_{\ \mathrm{static\ SU(2)}}$ are obtained by multiplying theirs by $\lambda = \frac{(xc_{23}-yc_{12})^2}{b_{24}(c_{12}^2+x^2)}$. Apart from this nor-

malization, we manage to recover their solution with:[19]

$$\tau_2^2 = -\frac{c_{23}}{\lambda}, \qquad \tau_2^1 = -\frac{2xyc_{23}+c_{12}c_{23}^2-y^2c_{12}}{(c_{12}^2+x^2)\lambda}, \tag{4.50}$$

$$\tau_3^1 = \frac{-c_{23}b_{45}(c_{12}^2+x^2)-2xyb_{25}c_{23}+(y^2-c_{23}^2)b_{25}c_{12}}{b_{24}(c_{12}^2+x^2)\lambda},\ \tau_3^2 = -\frac{b_{45}c_{12}+b_{25}c_{23}}{b_{24}\lambda}, \tag{4.51}$$

$$t_{1s} = \frac{c_{12}^2+x^2}{xc_{23}-yc_{12}}, \qquad b_s = -\frac{xy+c_{12}c_{23}}{xc_{23}-yc_{12}}. \tag{4.52}$$

Note that this λ is a part of the volume obtained in the limit (see (4.23)), hence it is well-defined and cannot be zero. Note also that we recover both their j_s and their Ω_{2s} with a factor λ difference, so that the normalization condition (2.12) stays correct for us and for them. As this normalization condition implies λ^2 we have the choice on the sign of the factor in j (we took $+\lambda$), which is related to the sign of b_{24}. It is then related to the sign of the t_i appearing.

Let us now look at the fluxes. We have only an F_3 as they do. In the simple case chosen for our parameters (4.24), by taking the limit of our $d(F_3)$, we exactly get theirs, multiplied by λ as it should be.

4.3.3 Limits of the third solution

We already mentioned that this solution was the T-dual of the second one. In [1], they also mention this point for the limit structures: the SU(3) limit of our solution (with $\alpha = 1$) consists of their "Model 3", and they mention that it is the T-dual to their "Model 2", which is the static SU(2) limit of our second solution, as just discussed. So by this T-duality argument, this SU(3) limit of our solution must match their "Model 3", and we will not consider further the SU(3) limit. Note for instance we get the "same" (T-dual) limit behaviours of our parameters: $b_{12} \sim xk_\perp$, $b_{23} \sim yk_\perp$ where x and y are real constants.

Let us now consider the static SU(2) limit of our third solution, which corresponds to the "Model 4" in [1]. With the same reasoning, it is probably the T-dual to the SU(3) limit of our second solution, that did not match to any solution found in [1]. We first note that our z matches theirs, modulo a global i factor. This difference is due to a different phase convention for the $O6$. Let us look at the other forms. As usual, considering the limit of $\mathrm{Re}(\Omega_2)_{||}$ and comparing it to j_s imposes $b_{25} \sim y_1k_{||}$, $b_{45} \sim y_2k_{||}$, $b_{24} \sim xk_{||}$ with y_1, y_2, x real constants. Once again, b_{24} going to zero can be allowed in this limit (see (4.35)). Using these behaviours of our parameters, we get the solution of [1] by taking the limit on our forms. In [1] they have:

$$\Omega_{2s} = (\tau_1^1 e^1 + \tau_2^1 e^3) \wedge \left(\tau_1^2 e^2 + \frac{\tau_2^1}{\tau_1^1}\tau_1^2 e^4 + \tau_3^2 e^5\right), \tag{4.53}$$

[19] Note that the complicated expressions for the parameters are related to the freedom left in choosing different expressions for the z^i (we did not take the same as them), as mentioned in subsection 4.1, and the t_i are different for the same reason.

JHEP08(2008)096

with complex parameters, and we match it and j_s with:

$$\tau_1^1 = \frac{b_{12}+ic_{12}}{\tau_1^2},\ \tau_2^1 = -\frac{b_{23}+ic_{23}}{\tau_1^2},\ \tau_3^2 = -\frac{\tau_1^2}{x}\left(y_2 + y_1\frac{(b_{23}+ic_{23})}{(b_{12}+ic_{12})}\right), \tag{4.54}$$

$$t_{1s} = \frac{(c_{23}b_{12}-b_{23}c_{12})|\tau_1^2|^2}{(b_{12}^2+c_{12}^2)x} = \frac{1}{t_{2s}}, \tag{4.55}$$

where τ_1^2 is not fixed. Note $t_{1s}t_{2s}=1$ is here the normalization condition (2.12).

Let us now look at the fluxes. There are slight differences, due to H and z. As explained previously, we do not get any H, while they do: this is an artefact of our procedure. Note nevertheless that their H is more constrained than it appears to be in [1], once one imposes it to be real. In our simple choice of parameters (4.36), by taking the limit on our $d(F_2)$, we get exactly theirs, modulo the factor coming from z (related to the difference between our z and theirs), and the following map we have to impose: $|\tau_1^2|^4 = c_{23}^2$. This last condition can seem surprising, but this difference is probably related to the absence of H in our limit. Besides, note that in this simplified choice, we get $y_1 = y_2 = \tau_3^2 = 0$, hence the solution obtained is clearly compatible with the sources appearing.

4.4 Some other interesting solutions?

In [1] they give the list of all the interesting nil/solvmanifolds and several information about each. Then they checked for each of these manifolds whether there were some SU(3) or static SU(2) solutions. They only found a few, and even less which were "new" (not T-dual to a warped T^6 with an $O3$). One can ask if we could do the same study for intermediate SU(2) structures. It would be a tedious job, so let us first make a few remarks. We showed that the intermediate SU(2) solutions we already found gave back the solutions found in [1] as limit solutions. But these were in [1] the only "new" solutions. So if there is any other intermediate SU(2) solution on one of the manifolds, there can only be two cases: either this solution has not any well defined limit, or it has but then the limit solution is not "new". As an example of the first case, we mention that there might be (to be verified) an intermediate SU(2) solution on n 5.2 with an orientifold along 56 which does not seem to have any well defined limit solution, because this set of manifold/orientifold does not appear in the list of solutions of [1]. As an example of the second case, let us mention that in [2] they find an intermediate SU(2) (not "new") solution on $n4.4$ and $n4.6$ of which some limit solution was found in [1] (and it was T-dual to a warped T^6). These remarks point out that to find quickly any other "new" intermediate SU(2) solution, we cannot use anymore the same intuition as before: trying to find some on the manifolds where "new" SU(3) or static SU(2) solutions were found in [1]. We have to use other ideas.

To find a "new" intermediate SU(2) solution, we will restrict our search to the specific case of solutions with several non completely overlapping orientifolds (in fact there cannot be more than two as we will see). The idea which leads us to do so is that it might be difficult to start with several non-overlapping sources, and get back by T-dualities a single $O3$. Furthermore, we choose the two non-overlapping sources to be orientifolds to use their projection properties: then, one can give some arguments which allow to discard some of the manifolds as candidates for intermediate SU(2) solutions. These arguments also help

JHEP08(2008)096

to understand why a second source was appearing in our solutions, while we were only imposing one. These (technical) arguments are given in appendix D. Starting with the whole list of possible manifolds and orientifolds, using these arguments (including symmetries of the algebra) we end up with the following restricted set of possible configurations of manifolds/O-planes (with couples of orientifolds between brackets):

$$\begin{aligned}
&\text{IIB (and } O5 \text{ sources) :}\\
&n3.3(45,16), n3.6(25,46), n3.9(25,46), n3.13(45,26)\\
&n3.14(45,26), n4.1(26,35), n4.2(26,35), n4.5(35,26), n4.6(35,26)\\
&s2.2(14,23), s2.4(14,25), s2.5(13,24), s2.6(14,23)\\
&s3.1(14,25)(15,24), s3.3(13,24), s4.1(14,25)\,, \qquad (4.56)\\
&\text{IIA (and } O6 \text{ sources) :}\\
&n3.9(235,346), n3.10(136,235), n3.11(136,235), n3.15(235,346), n3.16(136,235)\\
&n4.2(236,345), n4.3(146,345), n4.4(146,345), n4.6(246,345), n4.7(135,146)(135,236)\\
&s2.2(135,245), s2.5(136,246), s2.6(146,236), s3.2(146,256)\\
&s3.3(136,246), s3.4(145,246), s4.1(145,246)\,. \qquad (4.57)
\end{aligned}$$

The result is that we do not find any intermediate SU(2) solution with two non completely overlapping orientifolds on any of them[20] for IIB, apart from the previously found solutions: we tried the following configurations (manifold with the tried O-plane in brackets) without success:

$$\begin{aligned}
&n3.3(45), n3.6(46), n3.9(46), n3.13(45), n4.1(26), n4.2(26), n4.5(35), n4.6(35)\\
&s2.2(14), s2.4(14), s2.6(14), s3.1(14)(15), s3.3(13), s4.1(14)\,. \qquad (4.58)
\end{aligned}$$

For IIA, the work still has to be done.

5. Conclusion

In this paper, we have looked for "new" supersymmetric four-dimensional Minkowski flux vacua of type II string theory, with intermediate SU(2) structure. They are "new" in the sense they are not T-dual to a T^6 with an $O3$. We found three of them, in the large volume limit with smeared sources and constant parameters. Two of them are T-duals among themselves. To find these vacua, we introduced a new SU(2) structure, that transforms simply under the orientifold projection, and which actually corresponds to the SU(2) structure appearing with the dielectric pure spinors. Using these variables, we rewrote the projection conditions given in [2] in a more tractable way, and at the same time, the SUSY conditions became much simpler to solve. On the solutions found, we took the limit to the SU(3) or the static SU(2) cases, and recovered the solutions of [1], hence getting some intuition on what a dynamical SU(3) × SU(3) structure could look like.

[20] We notice that there might be a problem with the algebra of $s2.3$ given in [1], because it is supposed to have only two zeros according to its name, and it actually has three zeros. So we did not try anything on it.

Some points remain to be studied. One interesting point is the number $\mathcal{N}$ of four-dimensional SUSY preserved by the vacuum. Since these manifolds are parallelizable, the effective action is a priori maximally supersymmetric. Part of the supersymmetry can be broken by the presence of sources. Then, the vacuum can only preserve a fraction $\mathcal{N}$ of it. In [1] and [2], $\mathcal{N}$ was given in simple cases. Generically, looking at (2.8), one has to count the number of different pairs of internal spinors which are solutions to the SUSY conditions, and give the same vacuum. In other words, one has to count the number of different pairs of pure spinors which are solutions, and give the same metric and fluxes. It is the same as identifying the freedom left in a generic solution, which is for general solutions not an easy thing to do. That is why we did not discuss it in this paper, but a careful study could be interesting.

Another point is applying these techniques to study the possibility of AdS vacua with intermediate SU(2) structures. Actually, after the first appearance of this paper, it was shown in [22] that such solutions cannot exist.

A last point to study is the appearance of a second source in our solutions, while we were only imposing one. If we knew in advance that a second source was going to be present, this could have simplified the search in the case of an O-plane because of the other projection conditions to impose. In subsection 4.4, we discussed why the second O-plane could appear at the same time, but it is not clear whether its presence is necessary.

Finding these "new" solutions has several interests. It provides new examples of vacua on GCY, not related to the usual and widely studied T^6. It then gives some insight on new corners of the landscape, providing for instance new set-ups to compactify and find low energy effective actions. The compactification on these new manifolds has already been studied, and some arguments to find the effective actions have been given [23 – 26]. Note that finding first the four-dimensional effective action and then its vacuum has been proved to be equivalent to find directly the ten-dimensional vacuum on the product space-time the way we did here [25, 26].

Another possible interest is the link with non-geometrical backgrounds, as done in [1]. In particular, it was mentioned in [1] a possible link due to an asymmetric orbifold [27, 28]. These new solutions might provide new ingredients to understand it.

Finally, the formalism developed here could be interesting for dynamical solutions. Indeed, a dynamical solution would generically have the form of an intermediate SU(2) structure solution everywhere on the internal manifold, except at some points where it becomes an SU(3) structure (or a static SU(2) structure). In this paper we showed that the dielectric pure spinors and the associated SU(2) structure were the good variables in which to find intermediate SU(2) structure solutions, so they are probably the best variables to find dynamical solutions. But so far, despite the simplicity of the equations, our efforts in this direction have not met success.

An open side question concerns the search of a better discrimination of the manifolds on which to find a supersymmetric flux vacuum. Indeed, in [1], they looked among a long list of GCY and only found a few on which there was some vacuum. In the same way, in

JHEP08(2008)096

this paper, we tried to find vacua on some other manifolds without success. This seems to indicate the existence of some other refined criteria for which manifold to use, that we are missing at the moment. The mathematical specification of the manifolds is known: as explained, the manifolds should be a twisted GCY admitting an SU(3) × SU(3) structure, and being compatible with at least one O-plane. The existence of the SU(3) × SU(3) structure, and whether this structure is compatible with the orientifold projection, might be for instance criteria that haven't been implemented before beginning the search for vacua in [1] and in this paper. These could lead to a restricted set of manifolds/orientifolds.

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A. Conventions and derivation of background formulas

A.1 Some conventions of differential forms

In this appendix we give our conventions on (internal) gamma matrices, differential forms, some useful formulas about contractions, and conventions for the (six-dimensional) Hodge star.

We choose hermitian γ matrices (they are all purely imaginary and antisymmetric): $\gamma^{i\dagger} = \gamma^i$.

Here are some identities used (see [29] for more):

$$\begin{aligned} \{\gamma^m, \gamma^n\} &= 2g^{mn} & [\gamma^m, \gamma^n] &= 2\gamma^{mn} \\ \{\gamma^{mn}, \gamma^p\} &= 2\gamma^{mnp} & [\gamma^{mn}, \gamma^p] &= -4\delta^{p[m}\gamma^{n]} \\ \{\gamma^{mnpq}, \gamma^r\} &= 2\gamma^{mnpqr} & [\gamma^{mnpq}, \gamma^r] &= -8\delta^{r[m}\gamma^{npq]} \ . \end{aligned} \tag{A.1}$$

We take as a convention for a p-form A:

$$\gamma^{\mu_1 \dots \mu_p} \leftrightarrow dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p} \qquad A = \frac{1}{p!} A_{\mu_1 \dots \mu_p} \gamma^{\mu_1 \dots \mu_p} \ . \tag{A.2}$$

With some abuse in the notation, when we write the conjugate of a form expressed with real indices (i.e. on a real basis), we mean the conjugate of its components, hence for the one-form z appearing in the main part, we have (μ being a real index):

$$\overline{z} = \overline{z}_\mu \gamma^\mu \ . \tag{A.3}$$

For a p-tensor A, we define the antisymmetrization (with the $p!$ possible terms on the right-hand side) as:

$$A_{[\mu_1 \dots \mu_p]} = \frac{1}{p!}(A_{\mu_1\mu_2\mu_3\dots\mu_p} - A_{\mu_2\mu_1\mu_3\dots\mu_p} + A_{\mu_2\mu_3\mu_1\dots\mu_p} + \dots + A_{\mu_3\mu_4\mu_1\mu_2\mu_5\dots\mu_p} + \dots) \ . \tag{A.4}$$

For a p-form A and q-form B, we have the convention:

$$\frac{1}{(p+q)!}(A\wedge B)_{\mu_1\dots\mu_{p+q}}=\frac{1}{p!q!}A_{[\mu_1\dots\mu_p}B_{\mu_{p+1}\dots\mu_{p+q}]}\ . \tag{A.5}$$

For a p-form A and a 1-form $b=b_i\gamma^i$, we define the contraction:

$$b\llcorner A=\frac{1}{p!}b^\nu A_{\mu_1\dots\mu_p}\ p\ \delta^{[\mu_1}_\nu\gamma^{\mu_2\dots\mu_p]}=\frac{1}{(p-1)!}b^{\mu_1}A_{\mu_1\dots\mu_p}\gamma^{\mu_2\dots\mu_p}\ . \tag{A.6}$$

For generic 1-form x, p-form A and q-form B, one has:

$$x\llcorner(A\wedge B)=(x\llcorner A)\wedge B+(-1)^p\ A\wedge(x\llcorner B)\ . \tag{A.7}$$

We now give the conventions for the Hodge star $*$, with a given metric g. We introduce the totally antisymmetric tensor ϵ by $\epsilon_{\mu_1\dots\mu_m}=+1/-1$ for $(\mu_1\dots\mu_m)$ being any even/odd permutation of $(1\dots m)$, and 0 otherwise. Then, the convention used for the Hodge star is:[21]

$$*(dx^{\mu_1}\wedge\dots\wedge dx^{\mu_k})=\frac{\sqrt{|g|}}{(n-k)!}(-1)^{(n-k)k}\ \epsilon^{\mu_1\dots\mu_k\ \mu_{k+1}\dots\mu_n}\ g_{\mu_{k+1}\nu_{k+1}}\dots g_{\mu_n\nu_n}\ dx^{\nu_{k+1}}\wedge\dots\wedge dx^{\nu_n}\ , \tag{A.8}$$

with n the dimension of the space, $|g|$ the determinant of the metric. In the eigenvector basis $(v^1,\dots,v^n)$, with diagonalized metric D, we get for a k-form:

$$*(v^{\mu_1}\wedge\dots\wedge v^{\mu_k})=(-1)^{(n-k)k}\ \frac{\epsilon_{\mu_1\dots\mu_n}}{\sqrt{|g|}}D_{\mu_{k+1}\mu_{k+1}}\dots D_{\mu_n\mu_n}v^{\mu_{k+1}}\wedge\dots\wedge v^{\mu_n}\ , \tag{A.9}$$

without any summation on μ_{k+1}, …, μ_n, as we took off the $(n-k)!$, i.e. these indices are fixed; the $\epsilon_{\mu_1\dots\mu_n}$ is then only there for a sign. Note for a p-form A_p, one has:

$$**A_p=(-1)^{(n-p)p}\ A_p=(-1)^{(n-1)p}\ A_p\ . \tag{A.10}$$

A.2 SU(2) structure conditions

In this appendix we derive in a specific way the SU(2) structure conditions given in subsection 2.2. We start by considering a globally defined spinor η_+: this gives an SU(3) structure which has the properties (2.10). Let us now assume there is some holomorphic globally defined one-form z, for which we recall $||z||^2=\overline{z}\llcorner z=z\llcorner\overline{z}=\overline{z}^\mu z_\mu=2$. One can then always define two-forms from it:

$$j=J-\frac{i}{2}z\wedge\overline{z}\qquad\Omega_2=\frac{1}{2}\overline{z}\llcorner\Omega_3\ . \tag{A.11}$$

Note that j is clearly real. We are going to show that these define an SU(2) structure (the one naturally embedded in the SU(3)) since they satisfy the conditions (2.12), (2.13), and (2.14).

[21] We take the same "awkward sign convention" as in [1], in order to use the same pure spinors SUSY equations and the same calibration of the sources.

Holomorphicity is defined with respect to the almost complex structure (see footnote 2). Then, one can always have an hermitian metric (its non-zero components have one index holomorphic and the other anthropomorphic). Using this metric and some holomorphicity arguments in six dimensions, we first get that $z \llcorner \Omega_3 = 0$, $z \llcorner z = \overline{z} \llcorner \overline{z} = 0$. Furthermore, we get that Ω_2 is holomorphic, and deduce the following structure conditions:

$$\Omega_2 \wedge \Omega_2 = 0 \,, \tag{A.12}$$

$$z \llcorner \Omega_2 = 0, \qquad \overline{z} \llcorner \Omega_2 = 0 \,. \tag{A.13}$$

Using the same arguments, we get that $z \wedge \Omega_3 = 0$, and using (A.7), we have: $0 = \overline{z} \llcorner (z \wedge \Omega_3) = 2\Omega_3 - z \wedge (\overline{z} \llcorner \Omega_3)$, hence

$$\Omega_3 = z \wedge \Omega_2 \,. \tag{A.14}$$

Let us now recover the structure conditions involving j. We get using (A.7): $z \llcorner (\frac{z \wedge \overline{z}}{2}) = -z$, $\overline{z} \llcorner (\frac{z \wedge \overline{z}}{2}) = \overline{z}$. We have (using our almost complex structure and real indices) $\overline{z} \llcorner J = i\overline{z}$, because

$$(\overline{z} \llcorner J)_\nu = \overline{z}^\mu J_{\mu\nu} = -J_{\nu\mu} \overline{z}^\mu = -J_\nu{}^\mu \, \overline{z}_\mu = -(-i)\overline{z}_\nu = i\overline{z}_\nu \,. \tag{A.15}$$

So we deduce from the definition of j the following structure conditions:

$$\overline{z} \llcorner j = 0, \; z \llcorner j = 0 \,. \tag{A.16}$$

Using $J \wedge \Omega_3 = 0$ and (A.14), we deduce $z \wedge j \wedge \Omega_2 = 0$, and using (A.7), we then get:

$$j \wedge \Omega_2 = 0 \,. \tag{A.17}$$

To recover the remaining structure condition (2.12), we express the equality $\frac{4}{3} J^3 = i\Omega_3 \wedge \overline{\Omega}_3$ in terms of z, j and Ω_2, and get $\frac{4}{3}(j + \frac{i}{2} z \wedge \overline{z})^3 = iz \wedge \overline{z} \wedge \Omega_2 \wedge \overline{\Omega}_2$. Then, using the previously derived properties, contracting last formula with z and then contracting with $\overline{z}$, we finally get:

$$2 \, j^2 = \Omega_2 \wedge \overline{\Omega}_2 \,. \tag{A.18}$$

Going back to $\frac{4}{3} J^3 = i\Omega_3 \wedge \overline{\Omega}_3$, one deduces with (A.18):

$$j^3 = 0 \,. \tag{A.19}$$

A.3 Details on the compatibility conditions

In subsection 2.3, we explained that we needed a pair of compatible pure spinors. We mentioned that the compatibility conditions were actually implied by a set of SU(2) structure conditions seen in subsection 2.2. We are going to prove this implication here. The SU(2) structure conditions involved are (A.12), (A.17), (A.18), and (A.19). We will use the formulas (2.26) for the pure spinors, which are valid for any structure (intermediate or static SU(2), SU(3)), hence this result is valid for any structure. We give the following useful formula for any p-form A_p and q-form B_q:

$$\lambda(A_p \wedge B_q) = (-1)^{pq} \lambda(A_p) \wedge \lambda(B_q) \,, \tag{A.20}$$

and we recall the compatibility conditions given in subsection 2.3 (with the Φ_i defined in (2.38)):

$$\left\langle \Phi_1, \overline{\Phi}_1 \right\rangle = \left\langle \Phi_2, \overline{\Phi}_2 \right\rangle \neq 0\,, \tag{A.21}$$

$$\langle \Phi_1, X \cdot \Phi_2 \rangle = \left\langle \overline{\Phi}_1, X \cdot \Phi_2 \right\rangle = 0,\ \forall\ X = (x, y)\ \in\ T \oplus T^*\ . \tag{A.22}$$

In the following, we will use the Φ_i defined in (2.38) for IIA, but note these conditions are actually independent of the theory, since they are only involving a generic pair of pure spinors.

Using (2.26) for the pure spinors, the first compatibility condition gives

$$z \wedge \overline{z} \wedge \left(2k_\perp^2 j^2 + k_{||}^2 \Omega_2 \wedge \overline{\Omega}_2 - 2k_{||} k_\perp j \wedge \mathrm{Re}(\Omega_2) \right) \neq 0\,, \tag{A.23}$$

$$k_{||}^2 \frac{4}{3} i j^3 + i k_{||} k_\perp j^2 \wedge \mathrm{Re}(\Omega_2) = 4 \frac{z \wedge \overline{z}}{||z||^2} \wedge \Bigg(j^2 (k_{||}^2 - k_\perp^2) + \frac{1}{2} \Omega_2 \wedge \overline{\Omega}_2 (k_\perp^2 - k_{||}^2) + 2j \wedge \mathrm{Re}(\Omega_2) k_{||} k_\perp \Bigg)\,. \tag{A.24}$$

One can see that imposing (A.17), (A.18) and (A.19), (A.24) is automatically satisfied. Only (A.23) remains to be satisfied; it corresponds to the volume form being non-zero.

Let us now focus on the second compatibility condition. Since this condition is valid for any X, it is sufficient to study it in the two different cases where $X = (x, 0)$ and $X = (0, y)$. Then let us first look at $X = (0, y)$ and the condition $\langle \Phi_1, X \cdot \Phi_2 \rangle = 0$. One gets:

$$y \wedge z \wedge \Omega_2 \wedge (k_{||} k_\perp \Omega_2 + (k_{||}^2 - k_\perp^2) j) = 0\ . \tag{A.25}$$

As (A.25) is valid for any y, we get:

$$z \wedge \Omega_2 \wedge (k_{||} k_\perp \Omega_2 + (k_{||}^2 - k_\perp^2) j) = 0\ . \tag{A.26}$$

If one imposes (A.12) and (A.17), (A.25) is automatically satisfied.

Let us now consider $X = (x, 0)$ and still $\langle \Phi_1, X \cdot \Phi_2 \rangle = 0$. Using (A.7) and the following useful formula valid $\forall x\ \in\ T,\ \forall n\ \epsilon\ \mathbb{N}^*$

$$x \llcorner j^n = n\ j^{(n-1)} \wedge (x \llcorner j)\,, \tag{A.27}$$

one gets the following top form in terms of $x \llcorner z$, $x \llcorner j$, and $x \llcorner \Omega_2$:

$$(x \llcorner z) \left(\frac{i}{2} \Omega_2 \wedge j^2 + \frac{z \wedge \overline{z}}{2} \Omega_2 \wedge (-k_{||} k_\perp \Omega_2 + j(k_\perp^2 - k_{||}^2)) \right) - z \wedge \left(i k_\perp^2 \Omega_2 \wedge j \wedge (x \llcorner j) + \frac{k_{||}^2}{2} j^2 \wedge (x \llcorner \Omega_2) \right)\ . \tag{A.28}$$

Apart from the term in $j^2 \wedge (x \llcorner \Omega_2)$, the previous expression is obviously zero when one imposes (A.12) and (A.17). Using (A.7) and (A.27), one has

$$x \llcorner (j^2 \wedge \Omega_2) = 2\ j \wedge x \llcorner (j) \wedge \Omega_2 + j^2 \wedge (x \llcorner \Omega_2)\ . \tag{A.29}$$

JHEP08(2008)096

Hence the term in $j^2 \wedge (x \llcorner \Omega_2)$ is also zero when using (A.17), so the whole expression vanishes with (A.12) and (A.17). Thus, $\langle \Phi_1, X \cdot \Phi_2 \rangle = 0$ is automatically satisfied for any X when (A.12) and (A.17) are imposed.

One can play the same game with the condition $\left\langle \overline{\Phi}_1, X \cdot \Phi_2 \right\rangle = 0$. For $X = (0, y)$, one gets:

$$y \wedge z \wedge \left(k_{||} k_\perp (\Omega_2 \wedge \overline{\Omega}_2 - 2\, j^2) + j \wedge (k_{||}^2 \Omega_2 - k_\perp^2 \overline{\Omega}_2) \right) = 0\,, \tag{A.30}$$

which is obviously satisfied by imposing (A.17) and (A.18). For the $X = (x, 0)$ case, one gets:

$$\begin{aligned}(x \llcorner z) \Bigg(& - \frac{4i}{3} k_{||} k_\perp j^3 + \frac{i}{2} j^2 \wedge (k_{||}^2 \Omega_2 - k_\perp^2 \overline{\Omega}_2) - \frac{z \wedge \overline{z}}{2} \wedge (k_{||} k_\perp (\Omega_2 \wedge \overline{\Omega}_2 - 2 j^2) \\ & + j \wedge (k_{||}^2 \Omega_2 - k_\perp^2 \overline{\Omega}_2)) \Bigg) + z \wedge \left(i k_\perp (x \llcorner j) \wedge j \wedge (2 k_{||} j + k_\perp \overline{\Omega}_2) - \frac{k_{||}^2}{2} j^2 \wedge (x \llcorner \Omega_2) \right) = 0\,. \end{aligned} \tag{A.31}$$

Using the same kinds of tricks as before ((A.19) gives $j^2 \wedge (x \llcorner j) = 0$), we get that (A.17), (A.18) and (A.19) imply that the whole expression is zero. Thus, $\left\langle \overline{\Phi}_1, X \cdot \Phi_2 \right\rangle = 0$ is automatically satisfied for any X when (A.17), (A.18) and (A.19) are imposed.

We add the following point referring to subsection B.1: (A.24) can be rewritten in terms of the projection basis variables. It gives an equation which can be decomposed in the two following equations after projection:

$$\begin{aligned} & \mathrm{Re}(\Omega_2)_{||}^3 \frac{\gamma^3}{6} (3 + r(3 + 2k_{||}^2)) - \mathrm{Re}(\Omega_2)_\perp^2 \wedge \mathrm{Re}(\Omega_2)_{||} \frac{1}{2\gamma} (1 - r(3 + 2k_{||}^2)) \\ & \qquad + \frac{8}{||z||^2} \frac{1 + r}{2} \mathrm{Re}(z) \wedge \mathrm{Im}(z) \wedge \left(\frac{\mathrm{Re}(\Omega_2)_{||}^2}{1 - \cos(2\phi)} - \frac{\mathrm{Re}(\Omega_2)_\perp^2}{1 + \cos(2\phi)} \right) = 0\,, \end{aligned} \tag{A.32}$$

$$\begin{aligned} & \mathrm{Re}(\Omega_2)_\perp^3 \frac{1}{6\gamma^3} (3 - r(3 + 2k_{||}^2)) - \mathrm{Re}(\Omega_2)_{||}^2 \wedge \mathrm{Re}(\Omega_2)_\perp \frac{\gamma}{2} (1 + r(3 + 2k_{||}^2)) \\ & \qquad + \frac{8}{||z||^2} \frac{1 - r}{2} \mathrm{Re}(z) \wedge \mathrm{Im}(z) \wedge \left(\frac{\mathrm{Re}(\Omega_2)_{||}^2}{1 - \cos(2\phi)} - \frac{\mathrm{Re}(\Omega_2)_\perp^2}{1 + \cos(2\phi)} \right) = 0\,. \end{aligned} \tag{A.33}$$

Actually, one can show that these two equations are automatically satisfied using (B.4) and (B.5) which are a rewriting of some SU(2) structure conditions, because each of the three terms is zero. So we recover the fact that (A.24) is automatically satisfied after imposing the SU(2) structure conditions.

B. Going to the projection basis

In subsection 3.3, we explained that the good variables to use were the projection basis:

$$\mathrm{Re}(z),\ \mathrm{Im}(z),\ \mathrm{Im}(\Omega_2),\ \mathrm{Re}(\Omega_2)_{||},\ \mathrm{Re}(\Omega_2)_\perp,\ (j_{||},\ j_\perp)\,, \tag{B.1}$$

where $j_{||},\ j_\perp$ can eliminated using the projection conditions (3.15). So in this appendix, we rewrite the different equations to be solved in terms of these variables.

B.1 SU(2) structure conditions

In this appendix we rewrite the SU(2) structure conditions implying the compatibility conditions (see appendix A.3), namely (A.12), (A.17), (A.18) and (A.19). To do so we also use the projection conditions (3.15). The SU(2) structure conditions (A.12) and (A.17) are equivalent to (for both theories):

$$\begin{aligned}
\mathrm{Im}(\Omega_2) \wedge \mathrm{Re}(\Omega_2)_{||} &= 0\,, && \text{(B.2)}\\
\mathrm{Im}(\Omega_2) \wedge \mathrm{Re}(\Omega_2)_{\perp} &= 0\,, && \text{(B.3)}\\
\mathrm{Re}(\Omega_2)_{||} \wedge \mathrm{Re}(\Omega_2)_{\perp} &= 0\,, && \text{(B.4)}\\
\mathrm{Re}(\Omega_2)_{||}^2 &= \frac{1}{\gamma^2}\mathrm{Re}(\Omega_2)_{\perp}^2\,, && \text{(B.5)}\\
\mathrm{Re}(\Omega_2)_{||}^2 + \mathrm{Re}(\Omega_2)_{\perp}^2 &= \mathrm{Im}(\Omega_2)^2\ . && \text{(B.6)}
\end{aligned}$$

We do not get any new condition from (A.18) and (A.19), which can be understood the following way: as discussed in subsection 3.3, z, $\mathrm{Im}(\Omega_2)$, $\mathrm{Re}(\Omega_2)_{||}$, $\mathrm{Re}(\Omega_2)_{\perp}$ defines, modulo a rescaling, a new SU(2) structure (obtained by a rotation from the previous one). And so it is natural [17] to have the five previous "wedge conditions", and only them.

We recall that this last set of conditions, together with the projection conditions, is then enough to get all the compatibility conditions except from (A.23). For instance, in appendix A.3, we rewrote the compatibility condition (A.24) in terms of the projection basis variables and show that it was automatically satisfied using (B.4) and (B.5). Using the last relations and the projection basis, we can also rewrite (A.23):

$$\begin{aligned}
\mathrm{Re}(z) \wedge \mathrm{Im}(z) \wedge \left(\mathrm{Im}(\Omega_2)^2 + \frac{1}{k_{||}^2}(\frac{1-r}{2}\mathrm{Re}(\Omega_2)_{||}^2 + \frac{1+r}{2}\mathrm{Re}(\Omega_2)_{\perp}^2)\right) \neq 0\,,\\
\Leftrightarrow \mathrm{Re}(z) \wedge \mathrm{Im}(z) \wedge \mathrm{Re}(\Omega_2)_{||}^2 \neq 0\ . \qquad \text{(B.7)}
\end{aligned}$$

B.2 SUSY conditions

We derive in this appendix the SUSY conditions, starting from (2.35), (2.36), and (2.37) and a general expressions for the pure spinors, and then explaining the various steps leading to the equations given in subsection 3.4.

B.2.1 SUSY conditions derivation

We first use the following general expressions for the pure spinors:

$$\begin{aligned}
\Phi_+ &= \frac{a\overline{b}}{8}N^2 e^{\frac{1}{||z||^2}z\wedge\overline{z}}(k_{||}e^{-ij} - ik_{\perp}\Omega_2)\,,\\
\Phi_- &= -\frac{ab}{8}\frac{\sqrt{2}}{||z||}N^2 z \wedge (k_{\perp}e^{-ij} + ik_{||}\Omega_2)\,, \qquad \text{(B.8)}
\end{aligned}$$

with a, b, $||z||$, $N = ||\eta_+||$ constant and non-zero, and $k_{||}$, $k_{\perp}$ constant, and without any further fixing. For IIA, we just choose ab real (as it is the case when fixing further) and for IIB, we choose $a\overline{b}$ real, as it is the case for the $O5$ projection. We recall that the fluxes

are real. We then get the following equations, where (2.35) has been decomposed under its real and imaginary parts:

$$
\begin{aligned}
\text{IIA}: \quad & F_6 = 0 \\
& d(e^{2A-\phi})k_{||} = 0 \\
& d(e^{A-\phi}\mathrm{Re}(z))k_\perp = 0 \\
& d(e^{3A-\phi}\mathrm{Im}(z))k_\perp = -\frac{e^{4A}||z||}{\sqrt{2}abN^2} * F_4 \\
& d(e^{2A-\phi}\mathrm{Im}(\Omega_2))k_\perp = e^{2A-\phi}k_{||}H \\
& d\left(e^{2A-\phi}\left(-k_{||}j - k_\perp \mathrm{Re}(\Omega_2) + k_{||}\frac{z\wedge\overline{z}}{i||z||^2}\right)\right) = 0 \\
& d(e^{A-\phi}(-k_{||}\mathrm{Re}(z)\mathrm{Im}(\Omega_2) - \mathrm{Im}(z)(-k_\perp j + k_{||}\mathrm{Re}(\Omega_2)))) = e^{A-\phi}k_\perp H\mathrm{Re}(z) \\
& d(e^{3A-\phi}(-k_{||}\mathrm{Im}(z)\mathrm{Im}(\Omega_2)+\mathrm{Re}(z)(-k_\perp j+k_{||}\mathrm{Re}(\Omega_2)))) \\
& \qquad\qquad -e^{3A-\phi}k_\perp H\mathrm{Im}(z) = \frac{e^{4A}||z||}{\sqrt{2}abN^2} * F_2 \\
& d\left(e^{2A-\phi}\left(-\frac{1}{2}k_{||}j^2 - \frac{z\wedge\overline{z}}{i||z||^2}(-k_{||}j - k_\perp\mathrm{Re}(\Omega_2))\right)\right) = e^{2A-\phi}k_\perp H\wedge\mathrm{Im}(\Omega_2) \\
& d\left(e^{2A-\phi}\frac{z\wedge\overline{z}}{i||z||^2}\wedge\mathrm{Im}(\Omega_2)\right)k_\perp = e^{2A-\phi}H\left(-k_{||}j - k_\perp\mathrm{Re}(\Omega_2) + k_{||}\frac{z\wedge\overline{z}}{i||z||^2}\right) \\
& \frac{1}{2}k_\perp\mathrm{Re}(z)\wedge d(j^2) = H(-k_{||}\mathrm{Re}(z)\mathrm{Im}(\Omega_2) - \mathrm{Im}(z)(-k_\perp j + k_{||}\mathrm{Re}(\Omega_2))) \\
& d\left(e^{3A-\phi}\left(-\frac{1}{2}j^2\wedge\mathrm{Im}(z)\right)\right)k_\perp - e^{3A-\phi}H\wedge(-k_{||}\mathrm{Im}(z)\mathrm{Im}(\Omega_2) \\
& \qquad\qquad +\mathrm{Re}(z)(-k_\perp j+k_{||}\mathrm{Re}(\Omega_2))) = -\frac{e^{4A}||z||}{\sqrt{2}abN^2} * F_0\,, \qquad \text{(B.9)}
\end{aligned}
$$

$$
\begin{aligned}
\text{IIB}: \quad & F_5 = 0 \\
& d(e^{A-\phi})k_{||} = 0 \\
& d(e^{2A-\phi}\mathrm{Re}(z))k_\perp = 0 \\
& d(e^{2A-\phi}\mathrm{Im}(z))k_\perp = 0 \\
& d(e^{A-\phi}\mathrm{Im}(\Omega_2))k_\perp = e^{A-\phi}k_{||}H \\
& d\left(e^{3A-\phi}\left(-k_{||}j - k_\perp\mathrm{Re}(\Omega_2) + k_{||}\frac{z\wedge\overline{z}}{i||z||^2}\right)\right) = -\frac{e^{4A}}{a\bar{b}N^2} * F_3 \\
& d(e^{2A-\phi}(-k_{||}\mathrm{Re}(z)\wedge\mathrm{Im}(\Omega_2) - \mathrm{Im}(z)\wedge(-k_\perp j+k_{||}\mathrm{Re}(\Omega_2)))) = e^{2A-\phi}k_\perp H\wedge\mathrm{Re}(z) \\
& d(e^{2A-\phi}(\mathrm{Re}(z)\wedge(-k_\perp j+k_{||}\mathrm{Re}(\Omega_2)) - k_{||}\mathrm{Im}(z)\wedge\mathrm{Im}(\Omega_2))) = e^{2A-\phi}k_\perp H\wedge\mathrm{Im}(z) \\
& d\left(e^{A-\phi}\left(-\frac{1}{2}k_{||}j^2 + \frac{z\wedge\overline{z}}{i||z||^2}(k_{||}j + k_\perp\mathrm{Re}(\Omega_2))\right)\right) = e^{A-\phi}k_\perp H\wedge\mathrm{Im}(\Omega_2) \\
& d\left(e^{3A-\phi}k_\perp\frac{z\wedge\overline{z}}{i||z||^2}\wedge\mathrm{Im}(\Omega_2)\right) - e^{3A-\phi}H\wedge\left(-k_{||}j - k_\perp\mathrm{Re}(\Omega_2) + k_{||}\frac{z\wedge\overline{z}}{i||z||^2}\right) = \frac{e^{4A}}{a\bar{b}N^2} * F_1 \\
& \frac{1}{2}k_\perp\mathrm{Re}(z)\wedge d(j^2) = H\wedge(-k_{||}\mathrm{Re}(z)\wedge\mathrm{Im}(\Omega_2) - \mathrm{Im}(z)\wedge(-k_\perp j + k_{||}\mathrm{Re}(\Omega_2))) \\
& \frac{1}{2}k_\perp\mathrm{Im}(z)\wedge d(j^2) = H\wedge(\mathrm{Re}(z)\wedge(-k_\perp j + k_{||}\mathrm{Re}(\Omega_2)) - k_{||}\mathrm{Im}(z)\wedge\mathrm{Im}(\Omega_2))\,. \qquad \text{(B.10)}
\end{aligned}
$$

Then, one goes further by fixing as usual the parameters ($a = \overline{b}$ and $b = ae^{i\theta}$, $N = ||\eta_+|| = 1$ and $||z||^2 = 2$), going to the large volume limit (see subsection 3.4), and assuming $k_{||}$, $k_\perp$ to be non-zero. The next step is to introduce the projection basis variables which are the good variables to use here (see subsection 3.3). Actually, one can notice that the corresponding linear combinations (see (3.17)) already appear in the previous equations, indicating the possible simplifications. One way to get them is to apply σ on the equations and then project on the parallel and orthogonal parts.[22] This is another projection after the projection on real and imaginary parts and it gives much simpler equations. Note we have in each case $\sigma(H) = -H$. Using furthermore the projection conditions (3.15), and (3.17), the SUSY conditions are simplified to:

$$
\begin{aligned}
\text{IIA}: \quad & d(\mathrm{Im}(z))k_\perp = -g_s * F_4 \\
& d(\mathrm{Im}(\Omega_2))k_\perp = k_{||} H \\
& -k_{||} d(\mathrm{Im}(z)) \wedge \mathrm{Im}(\Omega_2) + \frac{1}{k_{||}} d(\mathrm{Re}(\Omega_2)_{||}) \wedge \mathrm{Re}(z) - \frac{1}{k_\perp} H \wedge \mathrm{Im}(z) = g_s * F_2 \\
& d\left(-\frac{1}{2} j^2 \wedge \mathrm{Im}(z)\right) k_\perp - H \wedge \left(-k_{||} \mathrm{Im}(z) \wedge \mathrm{Im}(\Omega_2) + \frac{1}{k_{||}} \mathrm{Re}(z) \wedge \mathrm{Re}(\Omega_2)_{||}\right) = -g_s * F_0 \\
& \\
& d(\mathrm{Re}(z)) = 0 \\
& d(\mathrm{Re}(\Omega_2)_\perp) = k_{||} k_\perp \mathrm{Re}(z) \wedge d(\mathrm{Im}(z)) \\
& H \wedge \mathrm{Re}(z) = -\frac{k_\perp}{k_{||}} d(\mathrm{Im}(z) \wedge \mathrm{Re}(\Omega_2)_{||}) \\
& d(j_{||} \wedge j_\perp) = 0 \\
& -\frac{1}{2} k_{||} d(j_{||}^2 + j_\perp^2) + \frac{1}{k_\perp} \mathrm{Re}(\Omega_2)_\perp \wedge \mathrm{Re}(z) \wedge d(\mathrm{Im}(z)) = k_\perp H \wedge \mathrm{Im}(\Omega_2) \\
& d(\mathrm{Im}(\Omega_2) \wedge \mathrm{Re}(\Omega_2)_\perp) = 0 \\
& H \wedge \mathrm{Re}(z) \wedge \mathrm{Im}(\Omega_2) = -H \wedge \mathrm{Im}(z) \wedge \mathrm{Re}(\Omega_2)_{||}\,, \qquad \text{(B.11)} \\
\text{IIB}: \quad & d(\mathrm{Im}(\Omega_2))k_\perp = k_{||} H \\
& d(\mathrm{Re}(\Omega_2)_{||}) = k_\perp e^{i\theta} g_s * F_3 \\
& H \wedge \mathrm{Re}(\Omega_2)_{||} = k_\perp e^{i\theta} g_s * F_1 \\
& \\
& d(\mathrm{Re}(z)) = 0 \\
& d(\mathrm{Im}(z)) = 0 \\
& \mathrm{Re}(z) \wedge H = -\frac{k_\perp}{k_{||}} \mathrm{Im}(z) \wedge d(\mathrm{Re}(\Omega_2)_\perp)
\end{aligned}
$$

[22] To do so, one has to know that σ "commutes" with Re() and Im() (obvious), and more importantly, it commutes with $d()$, the exterior derivative, since the algebra of the manifold we consider has to be invariant under the projection (this is the same condition as the compatibility of the sources on this manifold, we will explain this in greater details with our solutions).

JHEP08(2008)096

$$
\begin{aligned}
&\mathrm{Im}(z)\wedge H=\frac{k_\perp}{k_{||}}\mathrm{Re}(z)\wedge d(\mathrm{Re}(\Omega_2)_\perp)\\
&d(j_{||}\wedge j_\perp)=0\\
&-\frac{1}{2}k_{||}d(j_{||}^2+j_\perp^2)-\frac{1}{k_\perp}\mathrm{Re}(z)\wedge\mathrm{Im}(z)\wedge d(\mathrm{Re}(\Omega_2)_{||})=k_\perp H\wedge\mathrm{Im}(\Omega_2)\\
&-\mathrm{Re}(z)\wedge H\wedge\mathrm{Im}(\Omega_2)=\mathrm{Im}(z)\wedge H\wedge\mathrm{Re}(\Omega_2)_\perp\\
&\mathrm{Im}(z)\wedge H\wedge\mathrm{Im}(\Omega_2)=\mathrm{Re}(z)\wedge H\wedge\mathrm{Re}(\Omega_2)_\perp\,.
\end{aligned}
\tag{B.12}
$$

The final steps to get the SUSY conditions (3.27) and (3.28) are the following. One can first use the property derived in the next subsection, namely that in IIA/IIB there cannot be any 6-form which is positive/negative under σ. This gives the automatic annihilation of the last equation of IIA and the two last equations of IIB, and the simplification of the definition of F_0 in IIA. Second, one can use the SU(2) structure conditions, namely (B.2) to (B.6), to get some more simplifications.

B.2.2 More use of the projection basis

In IIA/IIB we introduce on a six-dimensional manifold an $O6/O5$ plane. The 1-form basis used is $(e^1,\dots,e^6)$ and we choose the three/two internal dimensions of the $O6/O5$ along directions labeled e^i_+. The other three/four directions are labeled e^i_-. The $\pm$ are used in reference to the action of σ on these forms: $\sigma(e^i_\pm)=\pm e^i_\pm$. We then deduce that any i-form O_i can be decomposed naturally as $O_{i||}+O_{i\perp}$, which can only be written this way:

$$
\begin{aligned}
O_{1||}&=\sum_i c^i_+\ e^i_+\,,\\
O_{1\perp}&=\sum_i c^i_-\ e^i_-\,,\\
O_{2||}&=\sum_{i,j} c^{ij}_{||+}\ e^i_+\wedge e^j_+ + c^{ij}_{||-}\ e^i_-\wedge e^j_-\,,\\
O_{2\perp}&=\sum_{i,j} c^{ij}_{\perp}\ e^i_+\wedge e^j_-\,,\\
O_{3\perp}&=\sum_{i,j,k} c^{ijk}_{\perp+}\ e^i_+\wedge e^j_+\wedge e^k_- + c^{ijk}_{\perp-}\ e^i_-\wedge e^j_-\wedge e^k_-\,,\\
&\dots
\end{aligned}
\tag{B.13}
$$

We can now show very easily that some conditions are automatically satisfied, or simplified, because we only have a limited number of $e^i_\pm$ in each theory. Especially, one can say that in IIA/IIB there cannot be any 6-form which is positive/negative under σ, due to the number of $e^i_\pm$, and so we can get the automatic annihilation of some conditions. It is the case in the SUSY conditions given above.

C. Discussion of some normalization with calibrated smeared sources

In this appendix, we motivate the normalization condition (4.4). From the work done on calibrations of supersymmetric sources [20, 2], we know that a calibrated source wrapping

an internal k-dimensional cycle Σ (in a d-dimensional internal space M), taken in a configuration without any flux pulled-back on it or world-volume flux $\mathcal{F}$, should satisfy the following condition:

$$\mathrm{Im}(\Phi_2)|_\Sigma = \frac{|a|^2}{8}\sqrt{|det(P(G))|}\; d\sigma^1 \wedge \ldots \wedge d\sigma^k \,, \tag{C.1}$$

where σ^i are coordinates on Σ, $|det(P(G))|$ is the absolute value of the determinant of the pull-back on the source world-volume of the ten-dimensional metric G, and $\mathrm{Im}(\Phi_2)$ is restricted to its components on Σ. With our ansatz (2.1) for G, we get:

$$\mathrm{Im}(\Phi_2)|_\Sigma = \frac{|a|^2}{8} e^{4A} V_\Sigma \,, \tag{C.2}$$

where V_Σ is the volume form of Σ. Further, with our conventions and in the large volume limit, we get:

$$e^{3A-\phi}\mathrm{Im}(\Phi_2)|_\Sigma = \frac{1}{8g_s} V_\Sigma \,, \tag{C.3}$$

where $e^{3A-\phi}$ should be understood as taken in the large volume limit.

The literature on calibrations introduces a current j_Σ, defined in our conventions as (the Mukai pairing was defined in (2.31))

$$\int_M \langle j_\Sigma, f\rangle = \int_\Sigma f \,, \tag{C.4}$$

for a given form f of Σ, and so one can introduce the one, j_{Σ_i}, associated to $e^{3A-\phi}\mathrm{Im}(\Phi_2)|_{\Sigma_i}$ for a source i. This current is actually related to the source current appearing in the right-hand side of the BI. Indeed, we can write (up to some factors that we won't take into account)

$$(d - H\wedge)F = j_{\mathrm{Total}} = \sum_{sources\ i} Q_i\; j_{\Sigma_i} \,, \tag{C.5}$$

Q_i being considered as the RR charge. So j_{Σ_i} corresponds to the density current, and can be written roughly as:

$$j_{\Sigma_i} \approx \delta^{d-k}(\Sigma)\; * V_{\Sigma_i} \,, \tag{C.6}$$

i.e. as a δ function to localize the source in its transverse directions, times the volume orthogonal to the cycle. Actually, the definition (C.4) shows that a sign like the one given by $\lambda(f)$ is entering the game, because a Mukai pairing is used instead of a simple wedge product. Hence we choose[23]

$$j_{\Sigma_i} = \delta^{d-k}(\Sigma)\; * \lambda(V_{\Sigma_i}) \,. \tag{C.7}$$

The smearing of the source corresponds to the idea that the source is not localized anymore in the transverse directions, or in other words, one doesn't see the δ function

[23]Note that we could multiply this expression for j_{Σ_i} by $\frac{\int_{\Sigma_i} V_{\Sigma_i}}{\int_M V}$, a natural factor when considering (C.4), which would make j_{Σ_i} metric independent. This is one more example of positive factors which could be taken into account.

JHEP08(2008)096

anymore, and so in this case, we write:

$$(d - H\wedge)F = \sum_{sources\ i} Q_i V^i, \qquad j_{\Sigma_i} = V^i\,, \tag{C.8}$$

and we should now have

$$V^i = *\lambda(V_{\Sigma_i})\ . \tag{C.9}$$

Actually, one can show in our conventions that

$$\langle *\lambda(V_{\Sigma_i}), V_{\Sigma_i}\rangle\ = V\,, \tag{C.10}$$

(where V is the internal space volume form). Hence, using (C.3) and the last result, we get to the following normalization condition in the large volume limit and for smeared sources:

$$\left\langle V^i, e^{3A-\phi}\mathrm{Im}(\Phi_2)\right\rangle = \frac{1}{8g_s}\ V\ . \tag{C.11}$$

We conclude with two remarks. First, this normalization could be refined, to take into account some forgotten factors like those appearing in the BI. But all these factors are positive, so they are not changing the sign of the charges, which is what matters in the end. Second, there are several ways to show that

$$\int_{M_6}\left\langle V^i, e^{3A-\phi}\mathrm{Im}(\Phi_2)\right\rangle\ > 0\,, \tag{C.12}$$

either by (C.4), or by the derivation of the no-go theorem done this way in [1], hence the sign given by $\lambda(\ldots)$ is indeed needed.

D. Solutions with several O-planes

In this appendix, we are going to explain the arguments that allow to reduce the list of possible sets of manifolds/O-planes for an intermediate SU(2) solution with several (non completely overlapping) orientifolds, as explained in subsection 4.4. Let us first consider the case of a type IIB solution (with an $O5$-plane). Using the same notations as in appendix B.2.2, we introduce the natural notation for the e^i: $e^i_\pm$, defined by $\sigma(e^i_\pm) = \pm e^i_\pm$. For an $O5$-plane in a six-dimensional manifold, there are four e^i_- and two e^i_+. To have an $O5$ source, it must first be compatible with the algebra of the manifold. The list given in [1] indicate what are the possible compatible O-planes, so it must be part of it. Furthermore, one should have a non-trivial F_3 to see this source appear, and we recall that the BI for that flux ($d(F_3)$) gives the co-volume of this source.[24] This co-volume is nothing but the wedge product of the four e^i_-. So there must be in the algebra of the manifold an e^k such that $d(e^k) = e^i_- \wedge e^j_-$. This is actually a non-trivial requirement: we have to look for manifolds which have two O-planes being listed in the compatible O-planes, and satisfying this co-volume requirement. For instance, the nilmanifold $n4.4$ has the following algebra:

[24] Note we consider here the simple case where there is no $-H \wedge F_1$ term, so the reasoning might not be the most general one. In IIA for F_2, we will not have this restriction since $-H \wedge F_0$ is an exact term.

$(0,0,0,0,12,14+23)$. The $O5$ compatible are along 56, 13, and 24. 56 is the only one satisfying the co-volume requirement, so there can be at most one O-plane on this manifold. Note that solutions with an O-plane along 56 are actually found in [1] and [2]. Doing this systematic check, some manifolds are excluded.

We can add other criteria on the O-planes. For a type IIB solution, we have $\sigma(z) = -z$. So the O-planes must be orthogonal to it (the "z criteria"). The z contains at least two distinct directions (otherwise $z \wedge \overline{z}$ would be zero which is forbidden) so the O-planes must be orthogonal to both. Furthermore, we have the SUSY condition $d(z) = 0$, hence the O-planes must be at least orthogonal to two of the 0 directions of the algebra. This "z criteria" allows to discard all the n $2.p$ nilmanifolds for instance, and also all the s $1.p$ solvmanifolds.

There is another important criteria. Let us use the following notation: z is at least along e^1_- and e^2_-, there are two other $-$ directions noted e^3_- and e^4_-, and the two $+$ directions are noted e^1_+ and e^2_+, with respect to the first O-plane. Each O-plane has to be orthogonal to z, so we can use the same notation for the second one: z is at least along e^1_- and e^2_- which are $-$ directions for the second O-plane too. To have an intermediate SU(2) structure solution, we must have a non-zero $\mathrm{Re}(\Omega_2)_{||}$ (see the conditions). As explained in appendix B.2.2, it is clear that $\mathrm{Re}(\Omega_2)_{||}$ only has components on two-forms $e^i_+ \wedge e^j_+$ or $e^i_- \wedge e^j_-$ but not on mixed $+$ and $-$. Because of the volume form condition given in (B.7), and because z is along $-$ directions, $\mathrm{Re}(\Omega_2)_{||}$ is in (B.7) the only form which can bring the e^i_+, so the pair (e^1_+, e^2_+) has to be present in the decomposition of $\mathrm{Re}(\Omega_2)_{||}$. For the solution to be compatible with both O-planes, $\mathrm{Re}(\Omega_2)_{||}$ must be "parallel" under both projections, so it means that the pairs (e^1_+, e^2_+) and (e^i_-, e^j_-) of one O-plane must corresponds to such pairs for the other O-plane, and not to mixed $+$ and $-$ pairs. More precisely, as we do not want the O-planes to be completely overlapping, so not along the same two directions, we deduce that the pair (e^1_+, e^2_+) for the first O-plane must be a (e^k_-, e^l_-) pair for the second and vice-versa. But the two O-planes already share two $-$ directions given by z: e^1_- and e^2_-. So (e^1_+, e^2_+) of the first O-plane corresponds to the pair (e^3_-, e^4_-) of the second and vice-versa. This just means the following "direction criteria": the two O-planes have to be along completely different directions, they cannot share one direction. These different directions are orthogonal to the z directions, so not much possibility is left: in particular, z is then "only" along two directions, and at most two non completely overlapping $O5$ are possible at the same time in an intermediate SU(2) solution. This is exactly the case for our solutions: for instance for the first solution, the O-planes are along 45 and 26, and z is along 1 and 3.

Applying carefully all these criteria, we find that the only remaining candidates are (with the a priori allowed directions of the O-planes in brackets):

$$
\begin{aligned}
&n3.3(45,16), n3.6(25,46), n3.9(25,46), n3.13(45,26)\\
&n3.14(45,26), n4.1(26,35,45), n4.2(26,35,45), n4.5\ (35,45,26,16), n4.6(35,26)\\
&s2.2(14,23), s2.4(14,15,24,25), s2.5(13,14,23,24), s2.6\ (14,23)\\
&s3.1(14,15,16,24,25,26), s3.3(13,14,23,24), s4.1\ (14,15,16,24,25,26)\ .
\end{aligned}
\tag{D.1}
$$

Of course we find our solutions among them. Note that only some couples of the O-planes indicated are possible. If one wants to find solutions to the list of condi-

tions (3.15), (B.2), (B.3), (B.4), (B.5), (B.6), (B.7) and (3.28), one can use some symmetry properties to avoid testing all the possibilities. For instance, $n4.2$ could a priori have a solution with the couples of O-planes $(26, 35)$ and $(26, 45)$. But its algebra, $(0, 0, 0, 0, 12, 15)$ is clearly symmetric under the exchange of 3 and 4, so one can restrict the search to one of the two couples. The same goes for n 4.1 for instance by doing the change of variables ($e^3 \to e^4$, $e^4 \to e^3$, $e^2 \to -e^2$, $e^5 \to -e^5$, $e^6 \to -e^6$). In this way, the list of manifolds/couples of O-planes to test is limited to:

$$\begin{aligned}
&n3.3(45, 16), n3.6(25, 46), n3.9(25, 46), n3.13(45, 26)\\
&n3.14(45, 26), n4.1(26, 35), n4.2(26, 35), n4.5(35, 26), n4.6(35, 26)\\
&s2.2(14, 23), s2.4(14, 25), s2.5(13, 24), s2.6(14, 23)\\
&s3.1(14, 25)(15, 24), s3.3(13, 24), s4.1(14, 25).
\end{aligned} \tag{D.2}$$

Let us now consider the type IIA case, with $O6$ as sources. This gives in six dimensions three e^i_- and three e^j_+. One can actually use the same kind of criteria. The "co-volume" criteria works the same with a non-trivial F_2: there must be in the algebra of the manifold an e^k such that $d(e^k) = e^i_- \wedge e^j_-$. The "$z$ criteria" also works: $\mathrm{Re}(z)$ is parallel to the O-plane, and its derivative is 0, so the O-planes have to share at least one direction which gives a zero in the algebra. $\mathrm{Im}(z)$ has to be orthogonal to the O-planes so they have to share at least one e^i_-. We recall that $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ are both non-zero and give at least two directions otherwise the volume form would be zero. So for each O-plane remain two $+$ and two $-$ directions. Can they share them? Considering exactly the same argument as before with $\mathrm{Re}(\Omega_2)_{||}$, we get the following "direction criteria": the non completely overlapping O-planes share exactly one direction, the one given by $\mathrm{Re}(z)$, and no other. This leads once again to the fact that at most two non completely overlapping $O6$ are possible at the same time in an intermediate SU(2) solution. Applying all these criteria we get to the following reduced list:

$$\begin{aligned}
&n3.9(235, 346), n3.10(136, 235), n3.11(136, 235), n3.15\ (235, 346), n3.16(136, 235)\\
&n4.2(236, 246, 345), n4.3(146, 345), n4.4(146, 236, 345), n4.6(246, 345),\\
&n4.7(135, 146, 236, 245)\\
&s2.2(135, 245), s2.5(136, 146, 236, 246), s2.6(146, 236), s3.2\ (146, 145, 256, 356)\\
&s3.3(136, 146, 236, 246), s3.4(145, 246, 346), s4.1\ (145, 156, 256, 245, 146, 246).
\end{aligned} \tag{D.3}$$

Considering the symmetries, we get the following list to be tried:

$$\begin{aligned}
&n3.9(235, 346), n3.10(136, 235), n3.11(136, 235), n3.15\ (235, 346), n3.16(136, 235)\\
&n4.2(236, 345), n4.3(146, 345), n4.4(146, 345), n4.6\ (246, 345), n4.7(135, 146)(135, 236)\\
&s2.2(135, 245), s2.5(136, 246), s2.6(146, 236), s3.2\ (146, 256)\\
&s3.3(136, 246), s3.4(145, 246), s4.1(145, 246).
\end{aligned} \tag{D.4}$$

Of course we recover our solutions in these lists (they pass all the criteria).

JHEP08(2008)096

References

[1] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *A scan for new $N = 1$ vacua on twisted tori*, *JHEP* **05** (2007) 031 [hep-th/0609124].

[2] P. Koerber and D. Tsimpis, *Supersymmetric sources, integrability and generalized- structure compactifications*, *JHEP* **08** (2007) 082 [arXiv:0706.1244].

[3] M. Graña, *Flux compactifications in string theory: a comprehensive review*, *Phys. Rept.* **423** (2006) 91 [hep-th/0509003].

[4] J. Polchinski and A. Strominger, *New vacua for type II string theory*, *Phys. Lett.* **B 388** (1996) 736 [hep-th/9510227].

[5] T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152].

[6] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[7] P. Candelas, G.T. Horowitz, A. Strominger and E. Witten, *Vacuum configurations for superstrings*, *Nucl. Phys.* **B 258** (1985) 46.

[8] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *Supersymmetric backgrounds from generalized Calabi-Yau manifolds*, *JHEP* **08** (2004) 046 [hep-th/0406137].

[9] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *Generalised structures of $\mathcal{N} = 1$ vacua*, *JHEP* **11** (2005) 020 [hep-th/0505212].

[10] N.J. Hitchin, *Generalised Calabi-Yau manifolds*, *Quart. J. Math. Oxford Ser.* **54** (2003) 281 [math.DG/0209099].

[11] M. Gualtieri, *Generalized complex geometry*, Ph.D. thesis, Oxford University [math.DG/0401221].

[12] S. Kachru, M.B. Schulz, P.K. Tripathy and S.P. Trivedi, *New supersymmetric string compactifications*, *JHEP* **03** (2003) 061 [hep-th/0211182].

[13] R. Minasian, M. Petrini and A. Zaffaroni, *Gravity duals to deformed SYM theories and generalized complex geometry*, *JHEP* **12** (2006) 055 [hep-th/0606257].

[14] N. Halmagyi and A. Tomasiello, *Generalized Kähler potentials from supergravity*, arXiv:0708.1032.

[15] D. Lüst and D. Tsimpis, *Supersymmetric AdS_4 compactifications of IIA supergravity*, *JHEP* **02** (2005) 027 [hep-th/0412250].

[16] J.P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Supersymmetric AdS_5 solutions of type IIB supergravity*, *Class. and Quant. Grav.* **23** (2006) 4693 [hep-th/0510125].

[17] G. Dall'Agata, *On supersymmetric solutions of type IIB supergravity with general fluxes*, *Nucl. Phys.* **B 695** (2004) 243 [hep-th/0403220].

[18] C. Jeschek and F. Witt, *Generalised G_2-structures and type IIB superstrings*, *JHEP* **03** (2005) 053 [hep-th/0412280].

[19] I. Benmachiche and T.W. Grimm, *Generalized $N = 1$ orientifold compactifications and the Hitchin functionals*, *Nucl. Phys.* **B 748** (2006) 200 [hep-th/0602241].

JHEP08(2008)096

[20] P. Koerber, *Stable D-branes, calibrations and generalized Calabi-Yau geometry*, *JHEP* **08** (2005) 099 [hep-th/0506154];
L. Martucci and P. Smyth, *Supersymmetric D-branes and calibrations on general $N = 1$ backgrounds*, *JHEP* **11** (2005) 048 [hep-th/0507099];
P. Koerber and L. Martucci, *Deformations of calibrated D-branes in flux generalized complex manifolds*, *JHEP* **12** (2006) 062 [hep-th/0610044].

[21] S. Fidanza, R. Minasian and A. Tomasiello, *Mirror symmetric* SU(3)*-structure manifolds with NS fluxes*, *Commun. Math. Phys.* **254** (2005) 401 [hep-th/0311122].

[22] C. Caviezel et al., *The effective theory of type IIA AdS_4 compactifications on nilmanifolds and cosets*, arXiv:0806.3458.

[23] M. Graña, J. Louis and D. Waldram, *Hitchin functionals in $N = 2$ supergravity*, *JHEP* **01** (2006) 008 [hep-th/0505264]; $\mathrm{SU}(3) \times \mathrm{SU}(3)$ *compactification and mirror duals of magnetic fluxes*, *JHEP* **04** (2007) 101 [hep-th/0612237].

[24] A. Micu, E. Palti and G. Tasinato, *Towards Minkowski vacua in type-II string compactifications*, *JHEP* **03** (2007) 104 [hep-th/0701173].

[25] P. Koerber and L. Martucci, *From ten to four and back again: how to generalize the geometry*, *JHEP* **08** (2007) 059 [arXiv:0707.1038].

[26] D. Cassani and A. Bilal, *Effective actions and $N = 1$ vacuum conditions from* $\mathrm{SU}(3) \times \mathrm{SU}(3)$ *compactifications*, *JHEP* **09** (2007) 076 [arXiv:0707.3125].

[27] S. Hellerman, J. McGreevy and B. Williams, *Geometric constructions of nongeometric string theories*, *JHEP* **01** (2004) 024 [hep-th/0208174].

[28] A. Dabholkar and C. Hull, *Duality twists, orbifolds and fluxes*, *JHEP* **09** (2003) 054 [hep-th/0210209]; *Generalised T-duality and non-geometric backgrounds*, *JHEP* **05** (2006) 009 [hep-th/0512005].

[29] P. Candelas and D.J. Raine, *Spontaneous compactification and supersymmetry in $D = 11$ supergravity*, *Nucl. Phys.* **B 248** (1984) 415;
W. Mueck, *General (anti-)commutators of gamma matrices*, arXiv:0711.1436.

JHEP08(2008)096